1. Show that a finite union of bounded sets is bounded. Show that the intersection of an arbitrary number of bounded sets is either bounded or empty.

2. Show that if a metric space \((X, d)\) has the property that every bounded sequence converges, then \(X\) consists of only one point.

3. Say whether the following sequences converge, and find their limit if they do:
   a) \(a_n = x^{-n}\) in \(C(\frac{1}{3}, \frac{2}{3})\) (the continuous functions on the open interval \((\frac{1}{3}, \frac{2}{3})\)) with the uniform - sup - metric;
   b) \(a_n = e^{-nx}\) in \(C[0, 1]\) with the uniform metric.
   c) \(a_n = (\alpha_n, f(\alpha_n))\) in \(\mathbb{R}\) with the Euclidean metric, where \(\alpha_n\) is a convergent sequence in \(\mathbb{R}\) with limit \(\alpha\), and \(f : \mathbb{R} \to \mathbb{R}\) is continuous.

4. Determine the diameter of the set \({|z| < r}\) in \((\mathbb{C}, d)\) where
   \[d(z_1, z_2) = \frac{2|z_1 - z_2|}{\sqrt{1 + |z_1|^2} \sqrt{1 + |z_2|^2}}\]

   Repeat the exercise for the set \({|z - 1| < r}\).

5. Let \(\{f_n(x)\}\) be a sequence of functions, continuous on the closed interval \([a, b] \in \mathbb{R}\), which is a Cauchy sequence with respect to the uniform metric on \(C[a, b]\).
   a) Prove that the sequence converges pointwise on \([a, b]\).
   b) Prove that the function defined by these pointwise limits is continuous on \([a, b]\).

6. Show that the taxicab and Euclidean metrics are topologically equivalent on \(\mathbb{R}^2\).

7. Show that the metrics
   \[d_1(z_1, z_2) = |z_1 - z_2|\]
   and
   \[d_2 = \frac{2|z_1 - z_2|}{\sqrt{1 + |z_1|^2} \sqrt{1 + |z_2|^2}}\]
   are topologically equivalent on \(\mathbb{C}\).