NEW TUTORIAL 3.1 FOR MATH3402

Question 1. *Let* $\{a_n\}_{n=1}^{\infty}$ *be a Cauchy sequence in a metric space in* (X, d) *and let* $\{a_{i_k}\}_{n=1}^{\infty}$ *be a subsequence of* $\{a_n\}_{n=1}^{\infty}$*. Show that*

$$\lim_{n\to\infty} d(a_n, a_{i_n}) = 0.$$

Question 2. *Let* $\{a_n\}_{n=1}^{\infty}$ *be a Cauchy sequence in a metric space in* (X, d) *and let* $\{a_{i_n}\}_{n=1}^{\infty}$ *be a subsequence of* $\{a_n\}_{n=1}^{\infty}$ *converging to* $p \in X$*. Show that* $\{a_n\}_{n=1}^{\infty}$ *also converges to* p*.*

Question 3. *Let* $\{b_n\}_{n=1}^{\infty}$ *be a Cauchy sequence in a metric space* X*, and let* $\{a_n\}_{n=1}^{\infty}$ *be a sequence in* X *such that* $d(a_n, b_n) < \frac{1}{n}$ *for every* $n \in \mathbb{N}$*.*

(i) Show that $\{a_n\}_{n=1}^{\infty}$ *is also a Cauchy sequence in* X*.*

(ii)Show that $\{a_n\}_{n=1}^{\infty}$ *converges to* $p \in X$ *if and only if* $\{b_n\}_{n=1}^{\infty}$ *converges to* $p \in X$*.*

Question 4. *Let* A *be a subset of a metric space* (X, d)*. Then* $x \in X$ *is a point of accumulation (limit point) of* A *if and only if there is an infinite sequence* $\{a_n\}_{n=1}^{\infty}$ *with* $a_n \in A$ *which converges to* x*.*