ASSIGNMENT 4 FOR MATH3402 IN 2013

Due date: 31 May 2013.

PLEASE SUBMIT IT TO THE ASSIGNMENT BOX,
LEVEL FOUR, PRIESTLEY BUILDING #67

Question 1. *(3 marks)*

Let $(X, \mathcal{T})$ *be a topological space and consider the product space*

$$X \times X := \{(x, y) \in X \times X : x \in X \, and \; y \in X\}.$$

Show that the topological space $(X, \mathcal{T})$ *is Hausdorff if and only if the the diagonal set* D *of* $X \times X$ *is closed in the product topological space* $X \times X$*, where the diagonal set is*

$$D := \{(x, y) \in X \times X : x = y \in X\}$$

Question 2. *(2 marks)*

Let A *be a closed and bounded set in* $\mathbb{R}$ *such that* $A \subset [a, b]$*. Assume that* $[a, b]$ *is compact. Then you show that* A *is compact in* $\mathbb{R}$*.*

Question 3. *(2 marks)*

Let $f : \mathbb{R}^3 \to \mathbb{R}^3$ *be a function given by*

$$f(x) = \begin{pmatrix} \frac{1}{2} & 0 & \frac{1}{3} \\ 0 & \frac{1}{5} & 0 \\ -\frac{1}{3} & 0 & \frac{1}{2} \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix}, \quad \forall x = \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} \in \mathbb{R}^3.$$

Prove that f *is a contraction on* $\mathbb{R}^3$ *with the standard Euclidean metric.*

Question 4. *(3 marks)*

Let $C^1([0, 1])$ *be the space of all functions having continuous derivatives. For each* $f \in C^1([0, 1])$*, set*

$$\|f\| = \left(\int_0^1 (|f|^2 + |f'|^2) \, dx \right)^{1/2}.$$

Show that $\| \cdot \|$ *is a norm of the space of* $C^1([0, 1])$*.*

Typeset by $\mathcal{A}\mathcal{M}\mathcal{S}$-TEX