ASSIGNMENT 1 FOR MATH3402 IN 2013

Due date: 23 March 2013.

PLEASE SUBMIT IT TO THE ASSIGNMENT BOX,
LEVEL FOUR, PRIESTLEY BUILDING #67

Question 1. *(3 marks)*

Let X be a universe. Let $\{A_i\}_{i\in\Lambda}$ be a family of sets in X, where Λ is a set of index. Prove

$$X\setminus \cap_{i\in\Lambda} A_i = \cup_{i\in\Lambda}(X\setminus A_i),$$

where $X\setminus A_i$ denotes the complement set of A_i.

Question 2. *(3 marks) Let $f:\mathbb{R}\to\mathbb{R}$ be a continuous function. Let $\{a_n\}_{n=1}^{\infty}$ be a convergent sequence in $\mathbb{R}$ with $\lim_{n\to\infty} a_n = a$ and $f(a) > 0$. Prove that: (i) There is a positive integer N such that for all $n > N$, $f(a_n) \geq \frac{f(a)}{2}$.*

(ii) The sequence $\{\frac{1}{f(a_n)}\}$ converges to $\frac{1}{f(a)}$; i.e. $\lim_{n\to\infty} \frac{1}{f(a_n)} = \frac{1}{f(a)}$.

Question 3. *(4 marks)*

Let $f:\mathbb{R}\to\mathbb{R}$ be a function given by

$$f(x) = \begin{cases} x, & \text{if } x \text{ is a rational number} \\ -x, & \text{if } x \text{ is an irrational number.} \end{cases}$$

Show that $f(x)$ is not continuous at every point $x_0 \neq 0$.

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