Question 1. (4 marks)
Let $X$ be a universe. Let $A_1$ and $A_2$ be two subsets of $X$. Prove

$$X \setminus (A_1 \cap A_2) = (X \setminus A_1) \cup (X \setminus A_2),$$

where $X \setminus A$ denotes the complement set of $A$.

Question 2. (3 marks)
Let $f : \mathbb{R} \to \mathbb{R}$ be a function given by

$$f(x) = \begin{cases} x^2 + 1, & \text{if } x \neq 0 \\ 0, & \text{if } x = 0 \end{cases}$$

Show that $f(x)$ is not continuous at the point $x = 0$.

Question 3. (4 marks + 1 bonus mark)
Let $\{a_n\}_{n=1}^\infty$ and $\{b_n\}_{n=1}^\infty$ be two convergent sequence in $\mathbb{R}$ with

$$\lim_{n \to \infty} a_n = a \quad \text{and} \quad \lim_{n \to \infty} b_n = b,$$

where $b_n \neq 0$ and $b \neq 0$. Prove that the sequence $\{a_n/b_n\}$ converges to $\frac{a}{b}$, i.e.

$$\lim_{n \to \infty} \frac{a_n}{b_n} = \frac{a}{b}.$$ 

Question 4. (4 marks)
Let $S = C^0([0, 1]; \mathbb{R})$ denote the set of all continuous functions $f : [0, 1] \to \mathbb{R}$.
For two continuous functions $f, g \in S$, set

$$d(f, g) = \max_{0 \leq x \leq 1} |f(x) - g(x)|.$$ 

Prove $(S, d)$ is a metric space.