Practice q's: answers. MATH3401

(1) $\left|\frac{z-3}{z+3}\right| = 2 \iff |z-3|^2 = 4|z+3|^2$

$\iff (x-3)^2 + y^2 = 4(x+3)^2 + 4y^2 \iff (x+5)^2 + y^2 = 4^2$

$\iff |z+5| = 4$ circle, radius 4, centre -5.

(2) a) $6^{1/4}\,\mathrm{cis}\,\frac{3\pi}{8},\ 6^{1/4}\,\mathrm{cis}\,\frac{7\pi}{8},\ 6^{1/4}\,\mathrm{cis}\,\frac{11\pi}{8},\ 6^{1/4}\,\mathrm{cis}\,\frac{15\pi}{8}$

b) $2,\ 2\cancel{\left(-\frac{1}{2}+\frac{\sqrt{3}}{2}i\right)} = -1 + \sqrt{3}\,i,\ -1-\sqrt{3}\,i$

(3) $1+i,\ 1-2i$

(4) $w = -i\left(\frac{z+1}{z-1}\right)$

(5) $-1 \pm 2i$

(6) b) $-i\ln(\sqrt{2}+1) + (2k+\frac{1}{2})\pi,\ -i\ln(\sqrt{2}-1) + (2k-\frac{1}{2})\pi,\ k \in \mathbb{Z}$

(7) a) ∞ b) 2

(8) $u_x = 2x^2$, $v_y = 3y^2$, $u_y = v_x = 0$. C/R are satisfied only on the curve $x = \frac{3}{2}y^2$. In particular they are satisfied at $(0,0)$. They are not satisfied on any open set, so the function is nowhere analytic, so no contradiction.

(10) a) $5i/(z+2i)^2$ b) $(\sin 2(z^2+iz))\cdot(2z+i)$

(9) $e^{z^2} = e^{x^2-y^2+2ixy} = u+iv$, where

$u = e^{x^2-y^2}\cos 2xy,\quad v = e^{x^2-y^2}\sin 2xy$

$\Rightarrow u_x = 2xe^{x^2-y^2}\cos 2xy - 2ye^{x^2-y^2}\sin 2xy = v_y,$

$\& \ u_y = -2ye^{x^2-y^2}\cos 2xy - 2xe^{x^2-y^2}\sin 2xy = -v_x.$