INTERNAL STUDENTS ONLY

THE UNIVERSITY OF QUEENSLAND

CANDIDATES MUST UPLOAD THEIR COMPLETED EXAMINATIONS PROMPTLY

Mid Semester Examination, 10 April 2024

MATH3401/3901

Complex Analysis/ Advanced Complex Analysis

(2 Unit Course)

Time: 50 Minutes for working

No perusal time before examination begins

CREDIT WILL BE GIVEN ONLY FOR WORK WRITTEN ON THIS EXAMINATION SCRIPT.
FULL WORKING MUST BE SHOWN.
Use the back pages if the space provided is insufficient, and/or for rough working.
Answer **all** questions. Show all working. Questions carry the marks indicated. Credit will only be given for work written on this examination paper. Total marks are 100.
Check that this examination paper has 10 printed pages.
Calculators - Casio FX82 series or UQ approved (labelled) only.
One A4 page (single sided) of hand-written notes permitted.
By uploading your completed exam, you are confirming that you complied with the University's academic integrity guidelines in completing this exam, that all work is your own, that you obtained no assistance directly or indirectly from any source other than those listed as permitted.

FAMILY NAME (PRINT): ______________________

GIVEN NAMES (PRINT): ______________________

STUDENT NUMBER: | | | | | | – | | |

SIGNATURE: ______________________

EXAMINER'S USE ONLY			
QUESTION	MARK	QUESTION	MARK
1		3	
2		4	
TOTAL MARKS			

1. [**25 marks**] Find the fourth roots of -256, i.e., find all solutions of $z^4 + 256 = 0$ in $\mathbb{C}$. Express your answers in the form $x + iy$, with $x, y \in \mathbb{R}$.

(Question 1 continued).

2. Recall $\coth z = \cosh z / \sinh z$.

   a) Prove that $\coth^{-1} z = \dfrac{1}{2} \log\left(\dfrac{z+1}{z-1}\right)$, clearly indicating any restrictions on your domain. **[15 marks]**

   b) Find all solutions of $\coth z = i$ (express them in the form $x + iy$). **[10 marks]**

(Question 2 continued).

3. (a)[**15 marks**] Let $f(z) = |z|^2$.

(i) Find all points $z \in \mathbb{C}$ at which f satisfies the Cauchy-Riemann equations. (Hint: the set is non-empty).
(ii) Find all points $z \in \mathbb{C}$ at which f is differentiable (Hint: the set is non-empty). Make sure you justify your answer.
(iii) Show that f is nowhere analytic in $\mathbb{C}$.
(iv) Explain why there is no contradiction between your answers to (ii) and (iii).

(b)[**10 marks**] Let $f(z) = \sin z + \cos z$.

(i) Is f bounded on $\mathbb{C}$?
(ii) Is there an unbounded subset of $\mathbb{C}$ on which f is bounded?
Explain your answers. Note: no marks will be given for an answer without explanation, even if it is correct.

(Question 3 continued).

4. (a) [**18 marks**] Determine the Möbius transformation (viewed as a mapping on $\overline{\mathbb{C}}$) mapping 3 to 0, i to ∞, and 0 to $-3i$.

   (b) [**7 marks**] Suppose that a Möbius transformation $z \mapsto \dfrac{az+b}{cz+d}$ (viewed as a mapping on $\overline{\mathbb{C}}$) maps ∞ to 0. Can you say anything about any of the coefficients a, b, c and d?

extra working space

bonus extra working space