

Mid Semester Examination, 05 April, 2023

**MATH3401/3901****Complex Analysis/Advanced Complex Analysis**  
(Unit Courses)**Time: 50 Minutes** for working

No reading time before examination begins

**CREDIT WILL BE GIVEN ONLY FOR WORK WRITTEN ON THIS EXAMINATION SCRIPT.****FULL WORKING MUST BE SHOWN.**

Use the back pages if the space provided is insufficient, and/or for rough working.

Answer **all** questions. Questions carry the marks indicated, total marks are 100.**Check that this examination paper has 11 printed pages.**

You may make use of any lecture notes that you have made related to the course. This can include your own handwritten or typed notes from lectures, and annotated pdfs of in-class notes, but does not include assignment solutions or past exam solutions.

Calculators - Casio FX82 series or UQ approved (labelled) only.

By uploading your completed exam, you are confirming that you complied with the University's academic integrity guidelines in completing this exam, that all work is your own, that you obtained no assistance directly or indirectly from any source other than those listed as permitted.

FAMILY NAME (PRINT): \_\_\_\_\_

GIVEN NAMES (PRINT): \_\_\_\_\_

STUDENT NUMBER:

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SIGNATURE: \_\_\_\_\_

EXAMINER'S USE ONLY			
QUESTION	MARK	QUESTION	MARK
1		3	
2		4	
TOTAL MARKS			

MATH3401/3901 — Complex Analysis/Advanced Complex Analysis  
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1. (a) Prove that  $\sinh^{-1} z = \log(z + (z^2 + 1)^{1/2})$ . Note that here,  $w \mapsto w^{1/2}$  is double valued. [12 marks]
- (b) Find all solutions  $z \in \mathbb{C}$  of  $\sinh z = 4i$  (express them in the form  $x + iy$ ). [13 marks]

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(Question 1 continued).

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2. Let  $f(z) = x^2 + y^2 - 2ixy$  (where  $z = x + iy$ ). [25 marks]
- (i) Find all points  $z \in \mathbb{C}$  at which  $f$  satisfies the Cauchy-Riemann equations. (Hint: the set is non-empty).
  - (ii) Find all points  $z \in \mathbb{C}$  at which  $f$  is differentiable (Hint: the set is non-empty). Make sure you justify your answer.
  - (iii) Show that  $f$  is nowhere analytic in  $\mathbb{C}$ .
  - (iv) Explain why there is no contradiction between your answers to (ii) and (iii).

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(Question 2 continued).

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3. (a) Show that there holds:  $\lim_{z \rightarrow \infty} \frac{z^9 + 7z}{z^5 + 17z} = \infty$  [12 marks]

(b) Determine the Möbius transformation (viewed as a mapping on  $\overline{\mathbb{C}}$ ) mapping  $\infty$  to 0, 0 to  $-i$ , and 1 to  $\infty$ . [13 marks]

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(Question 3 continued).

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4. (a) Calculate  $\frac{d}{dz}(1+i)^z$ , explaining any restrictions you need to make for your answer to be valid. [12 marks]
- (b) Find all solutions of  $\sin z = \cosh 4$  in  $\mathbb{C}$ . Express your answers in the form  $x + iy$ . [13 marks]



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(Question 4 continued).

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extra working space

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bonus extra working space