FLEXIBLE/ EXTERNAL

THE UNIVERSITY OF QUEENSLAND

CANDIDATES MUST UPLOAD THEIR COMPLETED EXAMINATIONS PROMPTLY

Mid Semester Examination, 11 April, 2022

MATH3401/MATH7431

Complex Analysis

(2 Unit Course)

Time: 15 minutes for scanning/download/upload, 50 Minutes for working

No perusal time before examination begins

CREDIT WILL BE GIVEN ONLY FOR WORK WRITTEN ON THIS EXAMINATION SCRIPT.
FULL WORKING MUST BE SHOWN.
Use the back pages if the space provided is insufficient, and/or for rough working.
Answer **all** questions. Show all working. Questions carry the marks indicated. Credit will only be given for work written on this examination paper. Total marks are 100.
Check that this examination paper has 10 printed pages.
You may make use of any lecture notes that you have made related to the course. This can include your own handwritten or typed notes from lectures, and annotated pdfs of in-class notes, but does not include assignment solutions or past exam solutions.
Calculators - Casio FX82 series or UQ approved (labelled) only.
By uploading your completed exam, you are confirming that you complied with the University's academic integrity guidelines in completing this exam, that all work is your own, that you obtained no assistance directly or indirectly from any source other than those listed as permitted.

FAMILY NAME (PRINT): ____________________

GIVEN NAMES (PRINT): ____________________

STUDENT NUMBER: | | | | | | | | |

SIGNATURE: ____________________

EXAMINER'S USE ONLY			
QUESTION	MARK	QUESTION	MARK
1		3	
2		4	
TOTAL MARKS			

1. (a)[**12 marks**] Find all solutions of the equation $z^2 - 3iz - 2 = 0$. Express your answers in the form $x + iy$, with $x, y \in \mathbb{R}$.

   (b) [**13 marks**] Find all solutions of the equation $z^4 - 3iz^2 - 2 = 0$. Again, express your answers in the form $x + iy$, with $x, y \in \mathbb{R}$.

(Question 1 continued).

2. (a) [**15 marks**] Prove that $\tan^{-1} z = \frac{i}{2} \log\left(\frac{i+z}{i-z}\right)$, noting any restrictions on your domain.

   (b) [**10 marks**] Find all solutions $z \in \mathbb{C}$ of $\tan z = 1$ (express them in the form $x + iy$).

(Question 2 continued).

3. (a)[**15 marks**] For $z = x + yi \in \mathbb{C}$, let $f(z) = x^2 + y^3 i$.

(i) Find all points $z \in \mathbb{C}$ at which f satisfies the Cauchy-Riemann equations. (Hint: the set is non-empty).
(ii) Find all points $z \in \mathbb{C}$ at which f is differentiable (Hint: the set is non-empty). Make sure you justify your answer.
(iii) Show that f is nowhere analytic in $\mathbb{C}$.
(iv) Explain why there is no contradiction between your answers to (ii) and (iii).

Explain your answers. Note: no marks will be given for an answer without explanation, even if it is correct.

(b) [**10 marks**] Calculate $\frac{d}{dz}(1+i)^z$, explaining any restrictions you need to make for your answer to be valid.

(Question 3 continued).

4. (a) [**18 marks**] Determine the Möbius transformation (viewed as a mapping on $\overline{\mathbb{C}}$) mapping 2 to 0, i to ∞, and 0 to $-2i$.

   (b) [**7 marks**] Determine the Möbius transformation mapping 2 to 0, $5i$ to i, and $1+i$ to i, or explain why no such Möbius transformation exists.

extra working space

bonus extra working space