

SCHOOL OF MATHEMATICS AND PHYSICS

MATH3401

Tutorial Worksheet

Semester 1, 2024, Week 5

(1) Show that the limit of the function

$$f(z) = \left(\frac{z}{\bar{z}}\right)^2$$

as z tends to 0 does not exist.

Hint: Do this letting nonzero points $z = (x, 0)$ and $z = (x, x)$ approach the origin.

(2) Find $f'(z)$ when

(a) $f(z) = \frac{z-1}{2z+1}$, ($z \neq -1/2$);

(b) $f(z) = \frac{(1+z^2)^4}{z^2}$, ($z \neq 0$).

(3) Determine where $f'(z)$ exists and find its value when

(a) $f(z) = \frac{1}{z}$;

(b) $f(z) = x^2 + iy^2$.

(c) $f(z) = z \operatorname{Im}(z)$.

(4) Show that each of these functions is differentiable in the indicated domain of definition, and also find $f'(z)$:

(a) $f(z) = \frac{1}{z^4}$, $z \neq 0$;

(b) $f(z) = \sqrt{r}e^{i\theta/2}$, ($r > 0, \alpha < \theta < \alpha + 2\pi$).

(5) Show that each of these functions is nowhere analytic:

(a) $f(z) = xy + iy$;

(b) $f(z) = 2xy + i(x^2 - y^2)$;

(c) $f(z) = e^y e^{ix}$.