

SCHOOL OF MATHEMATICS AND PHYSICS

MATH3401/3901:
Tutorial Worksheet
Semester 1, 2024, Week 2

(1) Prove that multiplication of complex numbers is commutative.

(2) Simplify each of these to a real number:

$$(a) \frac{1+2i}{3-4i} + \frac{2-i}{5i}; \quad (b) \frac{5i}{(1-i)(2-i)(3-i)}.$$

(3) Use the properties of conjugates and moduli to show that

$$(a) \overline{\bar{z} + 3i} = z - 3i; \quad (b) i\bar{z} = -i\bar{z}; \quad (c) \left| (2\bar{z} + 5)(\sqrt{2} - i) \right| = \sqrt{3}|2z + 5|.$$

(4) Verify that $\sqrt{2}|z| \geq |\operatorname{Re}z| + |\operatorname{Im}z|$.

Suggestion: Reduce this inequality to $(|x| - |y|)^2 \geq 0$.

(5) Use de Moivre's formula to derive the following trigonometric identities:

$$(a) \cos 3\theta = \cos^3 \theta - 3 \cos \theta \sin^2 \theta; \quad (b) \sin 3\theta = 3 \cos^2 \theta \sin \theta - \sin^3 \theta.$$