MATH3401/3901: Complex Analysis/Advanced Complex Analysis

First Semester 2024
19/02/24

Assignment Number 1

Problem 1 *(3 points)* Graph the following regions in the complex plane:
a) $\{z\colon 2\mathrm{Re}\ z \leq (\mathrm{Im}\ z)^2\}$;
b) $\{z\colon \pi/4 < \mathrm{Arg}\ z \leq 3\pi/4\}$;
c) $\{z\colon |z-2i| \geq \pi\}$.

Problem 2 *(3 points)* Find *all* complex solutions of the following equations:
a) $\overline{z} = z$;
b) $\overline{z} + z = 0$;
c) $\overline{z} = 4/z$.

Problem 3 *(3 points)* Express the following in the form $x + iy$:
a) $\dfrac{i}{1-i} + \dfrac{1-i}{i}$; **b)** all the 3rd roots of $64i$; **c)** $\left(\dfrac{i+1}{\sqrt{2}}\right)^{1337}$.

Problem 4 *(5 points)* Recall the following (you do not have to prove it): For $n \in \mathbb{Z}_+$, $\sqrt{n} \in \mathbb{Q}$ if and only if n is a perfect square, i.e., $n = m^2$ for some $m \in \mathbb{Z}$.
Keeping this in mind, the set "$\mathbb{Q}$ adjoin $\sqrt{2}$" is defined by: $\mathbb{Q}(\sqrt{2}) = \{p+q\sqrt{2} : p, q \in \mathbb{Q}\}$.
a) Show that $\mathbb{Q}(\sqrt{2})$ is a *subfield* of $\mathbb{R}$ (i.e., it is a subset that is a field).
Hint 1: remember that $\mathbb{Q}$ and $\mathbb{R}$ are fields, and note that $\mathbb{Q}(\sqrt{2}) \subset \mathbb{R}$: hence you will get many, indeed, most of the properties almost free.
Hint 2: make sure you check closure.
b) Is $\sqrt{3} \in \mathbb{Q}(\sqrt{2})$? Explain your answer.

Problem 5 *(3 points)* A field $\mathbb{F}$ is called *ordered* if there exists a distinguished subset $\mathcal{P} \subset \mathbb{F}$, closed under addition and multiplication, such that the *trichotomy law* holds, i.e., For every $x \in \mathbb{F}$, precisely one of the following holds:
(i)$x \in \mathcal{P}$;
(ii)$x = 0$; or
(iii)$-x \in \mathcal{P}$.
(Note that we can then define an ordering via $x > y \Leftrightarrow x - y \in \mathcal{P}$).
Prove that $\mathbb{C}$ is not ordered. (Hint: first show that there must hold $-1 \notin \mathcal{P}$).

Problem 6 *(3 points)* Write one or at most two paragraphs on the *Schwarz-Christoffel transformation*. You should also include one relevant picture. Neither your paragraphs nor your picture should be from Wikipedia. Provide references as appropriate: see the UQ student guide at https://uq.pressbooks.pub/digital-essentials-write-cite-submit/chapter/module-overview/. Note also that you are not permitted to use ChatGPT or similar generative AI for this question, or indeed for any question on the assignment, as per the course profile.

(Extra style points if the answer is in verse. Disclaimer: style points are not exchangeable for actual assignment points.)

Due: 2:00 P.M., Friday, 08/03/2024.
Current assignments will be available at `http://www.maths.uq.edu.au/courses/MATH3401/Tutorials.html`