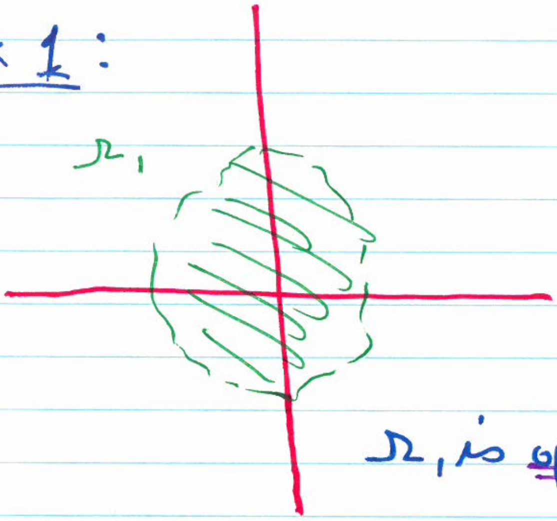
LECTURE 12

Ex 1:

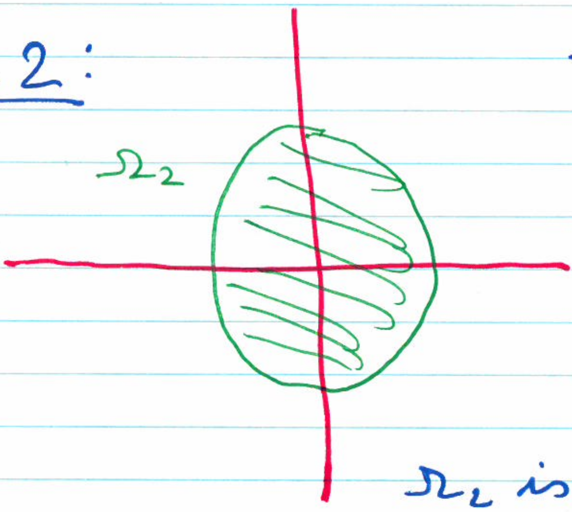


$\Omega_1 = B_1(0) = B_1 = B = \{z : |z| < 1\}$

* $\text{Int}\,\Omega_1 = \Omega_1$
* $\text{Ext}\,\Omega_1 = \{z : |z| > 1\}$
* $\partial\Omega_1 = \{z : |z| = 1\} = S^1$
* $\Omega_1^c = \{z : |z| \geq 1\}$.

Ω_1 is <u>open</u>

Ex 2:

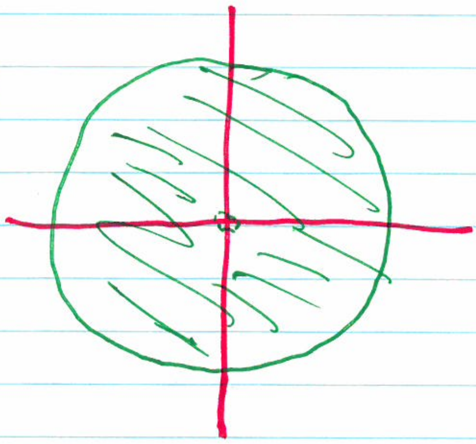


$\Omega_2 = \overline{B}_1(0) = \overline{B}_1 = \overline{B} = \{z : |z| \leq 1\}$

* $\text{Int}\,\Omega_2 = \Omega_1$
* $\text{Ext}\,\Omega_2 = \text{Ext}\,\Omega_1$
* $\partial\Omega_2 = S^1 = \partial\Omega_1$
* $\Omega_2^c = \text{Ext}\,\Omega_1$

Ω_2 is <u>closed</u>

Ex 3:



$\Omega_3 = \{z : 0 < |z| \leq 1\}$

$\text{Int}\,\Omega_3 = \{0 < |z| < 1\}$.

$\text{Ext}\,\Omega_3 = \text{Ext}\,\Omega_1$

$\partial\Omega_3 = S^1 \cup \{0\}$.

$\Omega_3^c = \text{Ext}\,\Omega_1 \cup \{0\}$

Ω_3 is neither open nor closed.

Note: Ω_1 is open, Ω_1^c is closed;
Ω_2 is closed, Ω_2^c is open.
Ω_3 & Ω_3^c are neither open nor closed.
Only clopen sets in $\mathbb{C}$ are $\emptyset$ & $\mathbb{C}$.

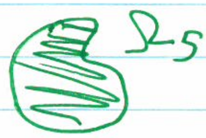
closed & open.

* $\Omega \subseteq \mathbb{C}$ is connected if there do not exist non-empty, disjoint open sets Ω' & Ω'' s.t.

$\Omega \subseteq \Omega' \cup \Omega''$ &

$\Omega' \cap \Omega \neq \emptyset$ & $\Omega'' \cap \Omega \neq \emptyset$.

Ω_4 is not connected, i.e., it is disconnected.



Ω_5 is connected, as are $\Omega_1, \Omega_2, \Omega_3$.

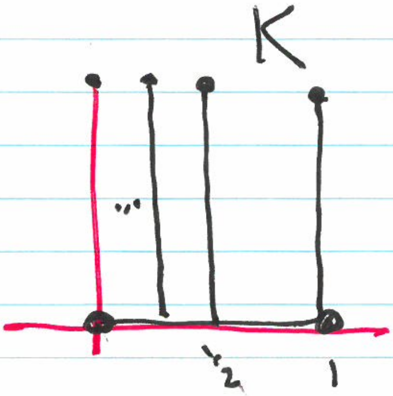
* $\Omega \subseteq \mathbb{C}$ is piecewise affinely path connected if any two points in Ω can be connected by a finite ~~#~~ of line segments in Ω, joined end-to-end.

For open sets in $\mathbb{C}$, the two definitions are equivalent.

In general, the two def^n's are not equivalent.

Consider $K = \bigcup_{n=1}^{\infty} \{ \frac{1}{n} + yi \mid 0 \leq y \leq 1 \}$

$\cup \{ x + 0i \mid 0 \leq x \leq 1 \} \cup \{ i \}$.



K is connected, but is not p.w. aff. p.c., as you can't get a path from i to any other point in K.

<u>Example</u>: if Ω_1 & Ω_2 are open in $\mathbb{C}$, then so is $\Omega_1 \cap \Omega_2$.

<u>Pf</u>: If $\Omega_1 \cap \Omega_2 = \emptyset$, we are done ($\emptyset$ is open).

Otherwise, for any $z \in \Omega_1 \cap \Omega_2$:

$z \in \Omega_1 \Rightarrow \exists\, \varepsilon_1 > 0$ s.t. $B_{\varepsilon_1}(z) \subset \Omega_1$, (✱)

& $z \in \Omega_2 \Rightarrow \exists\, \varepsilon_2 > 0$ s.t. $B_{\varepsilon_2}(z) \subset \Omega_2$ (☺),

since Ω_1 & Ω_2 are open.

So, set $\varepsilon = \underbrace{\min}_{\text{minimum}} \{ \varepsilon_1, \varepsilon_2 \}$: note $\varepsilon > 0$.

$B_\varepsilon(z) \subset \Omega_1$ by (✱) ; $B_\varepsilon(z) \subset \Omega_2$ by (☺).

So $B_\varepsilon(z) \subset \Omega_1 \cap \Omega_2$.

Since z was arbitrary in $\Omega_1 \cap \Omega_2$, there holds $\operatorname{Int}(\Omega_1 \cap \Omega_2) = \Omega_1 \cap \Omega_2 \Rightarrow \Omega_1 \cap \Omega_2$ is open $\square$

Next: limits