LECTURE 11

Recall $\sin^{-1} z = -i\log\left(iz + (1-z^2)^{1/2}\right)$

E.g., $\sin^{-1}(-i) = -i\log(1+2^{1/2})$ $\circledast$

$$2^{1/2} = \left(2e^{i\cdot 0}\right)^{1/2}$$

$$= \left\{\sqrt{2}\,e^{i\cdot 0/2},\ \sqrt{2}\,e^{i(0+2\pi)/2}\right\}$$

$$= \left\{\sqrt{2},\ \sqrt{2}\,e^{i\pi}\right\}$$

$$= \{\pm\sqrt{2}\}.$$

$\arg(1+\sqrt{2})$ $\shortparallel$

So, note $\log(1+\sqrt{2}) = \ln(1+\sqrt{2}) + 2n\pi i$, $n\in\mathbb{Z}$

& $\log(1-\sqrt{2}) = \ln\underbrace{(\sqrt{2}-1)}_{=|1-\sqrt{2}|} + (2n+1)\pi i$, $n\in\mathbb{Z}$

$$\sin^{-1}(-i) = \left\{-i\left[\ln(1+\sqrt{2}) + 2n\pi i \quad n\in\mathbb{Z}\right]\right\} \cup$$

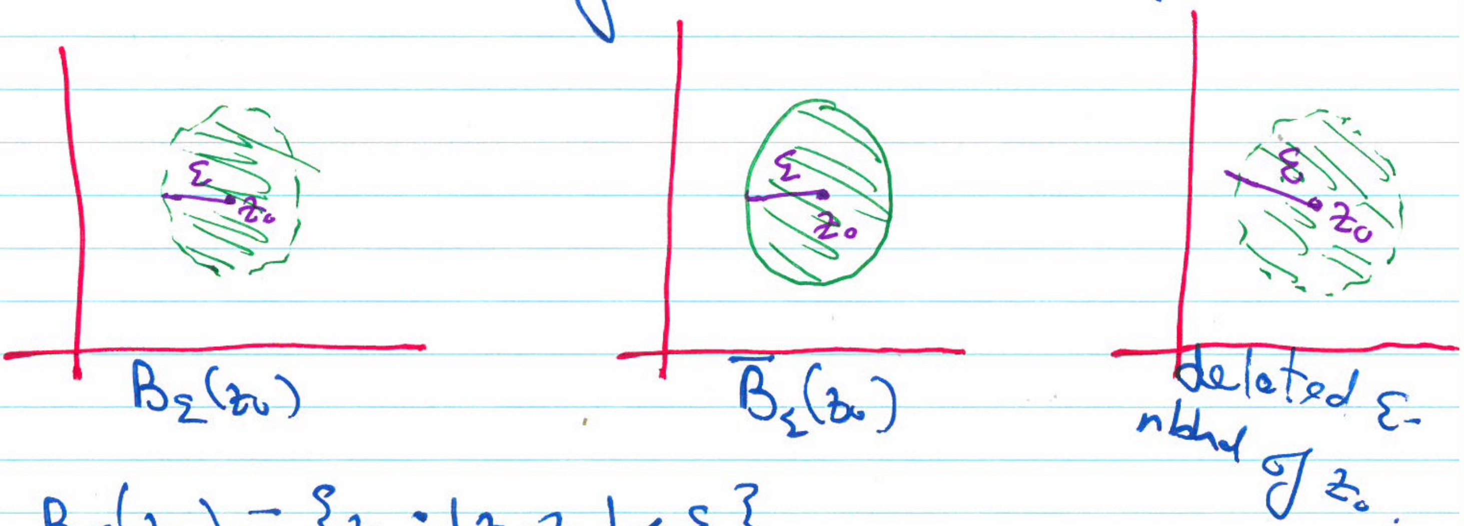
$$\left\{-i\left[\ln(\sqrt{2}-1) + (2n+1)\pi i \quad n\in\mathbb{Z}\right]\right\}.$$

$$= \left\{-i\ln(1+\sqrt{2}) + 2n\pi,\ n\in\mathbb{Z}\right\} \cup$$

$$\left\{-i\ln(\sqrt{2}-1) + (2n+1)\pi,\ n\in\mathbb{Z}\right\}.$$

§12 (§11 8Ed) Topology

* Given $z_0 \in \mathbb{C}$ & $\varepsilon > 0$, $B_\varepsilon(z_0)$ denotes the (open) ball of radius ε about z_0, a.k.a. ε-neighbourhood of z_0, given by $\boxed{\{z \in \mathbb{C} : |z - z_0| < \varepsilon\}}$
* $\overline{B}_\varepsilon(z_0)$ = closed ball of radius ε about z_0, a.k.a. closed ε-neighbourhood of z_0, given by $\boxed{\{z \in \mathbb{C} : |z - z_0| \le \varepsilon\}}$.
* Deleted ε-nbhd of z_0 : $\{z : 0 < |z - z_0| < \varepsilon\}$.

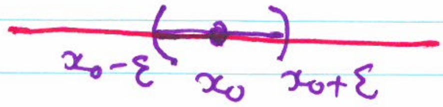


$$B_\varepsilon(z_0) = \{z : |z - z_0| < \varepsilon\}$$

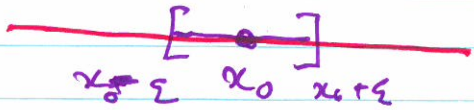
$$\begin{aligned}
|z - z_0| &= |(x+iy) - (x_0 + iy_0)| \\
&= |(x - x_0) + i(y - y_0)| \\
&= \sqrt{(x-x_0)^2 + (y-y_0)^2} \\
&= \|(x,y) - (x_0,y_0)\|_{\mathbb{R}^2} \\
&= d((x,y),(x_0,y_0))_{\mathbb{R}^2} \\
&= d(z, z_0)_{\mathbb{C}}.
\end{aligned}$$

$d(\cdot,\cdot)$ = distance.

In $\mathbb{R}$



$B_\varepsilon(x_0)$



$\overline{B_\varepsilon}(x_0)$

Take $\Omega \subseteq \mathbb{C}$:

* $z \in \mathbb{C}$ is an <u>interior point of Ω</u> if $\exists\, \varepsilon > 0$ s.t. $B_\varepsilon(z) \subset \Omega$. (Note: $\Rightarrow B_{\varepsilon'}(z) \subset \Omega \;\forall \varepsilon',\ 0<\varepsilon'<\varepsilon$)

such that

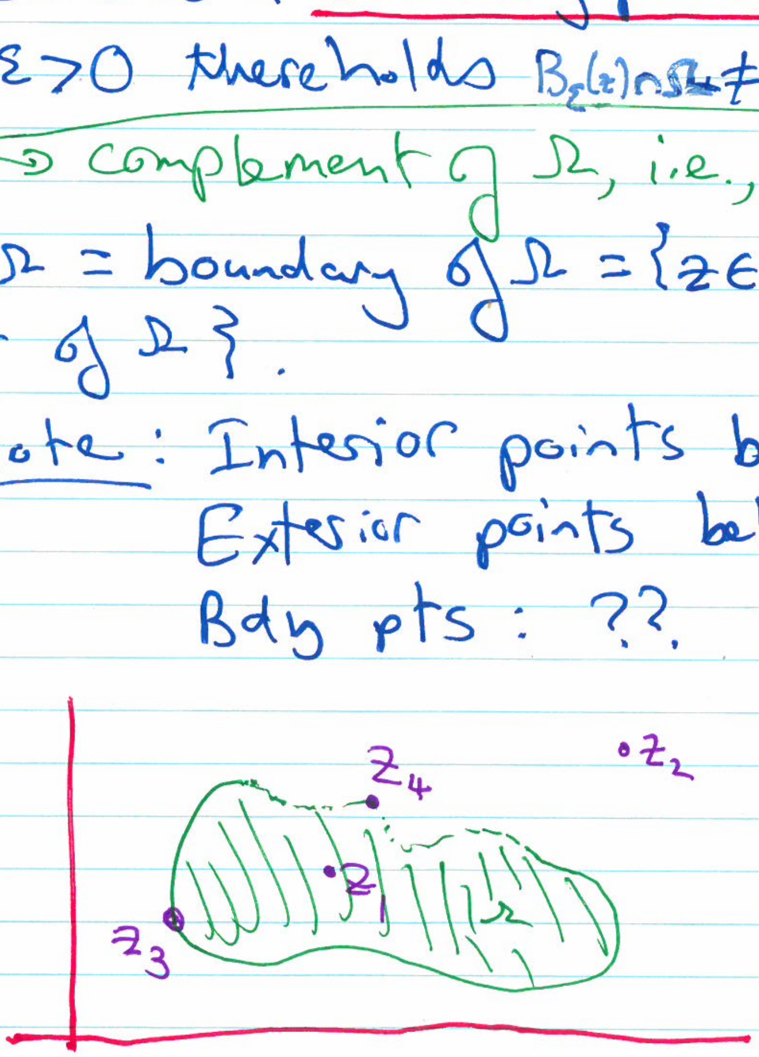
* $z \in \mathbb{C}$ is an <u>exterior point of Ω</u> if $\exists\, \varepsilon > 0$ s.t. $B_\varepsilon(z) \cap \Omega = \emptyset$.

* $z \in \mathbb{C}$ is a <u>boundary point of Ω</u>, <u>$z \in \partial\Omega$</u>, if $\forall\, \varepsilon > 0$ there holds $B_\varepsilon(z) \cap \Omega \neq \emptyset$, $B_\varepsilon(z) \cap \Omega^c \neq \emptyset$.

$\to$ complement of Ω, i.e., $\mathbb{C} \setminus \Omega$.

$\partial\Omega$ = boundary of Ω = $\{z \in \mathbb{C} : z \text{ is a bdy pt of } \Omega\}$.

<u>Note</u>: Interior points belong to Ω,
Exterior points belong to Ω^c
Bdy pts : ??



z_1 : interior pt.
z_2 : exterior pt.
z_3 : bdy pt, $z_3 \in \Omega$
z_4 : bdy pt, $z_4 \notin \Omega$.

* $\text{Int}\,\Omega$ = interior of Ω

  $= \{z : z \text{ is an interior point of } \Omega\}$

  $\text{Ext}\,\Omega$ = exterior of Ω

  $= \{z : z \text{ is an exterior point of } \Omega\}$.

* Ω is open if $\Omega = \text{Int}\,\Omega$.
* Ω is closed if $\partial\Omega \subseteq \Omega$.