Only clopen (both open & closed) sets in \( C \) are \( C \) & \( \emptyset \).

\[ \emptyset \subseteq C \text{ is connected if there do not exist open, disjoint sets } \emptyset \text{ and } \emptyset \text{ with } \emptyset \subseteq C \text{ and } \emptyset \cap \emptyset \neq \emptyset. \]

\[ S \subseteq S' \cup S'' \text{ and } S' \cap S \neq \emptyset \text{ and } S'' \cap S \neq \emptyset. \]

\[ S_4 \text{ is disconnected} \]

\[ S_5 \text{ is connected} \]
$\mathbb{R}$ is connected.

$\mathbb{R}$ is connected.

$\mathbb{R}^1$ connected, $\mathbb{R}^2$ disconnected, $\mathbb{R}^3$ connected.

$\mathbb{R}^2$ path-connected by points in $\mathbb{R}^2$ can be connected by a finite number of segments in $\mathbb{R}^2$.

$\mathbb{R}^1, \mathbb{R}^2$ disconnected, $\mathbb{R}^3$ connected.

$\mathbb{R}^2 \setminus \mathbb{B}_r(1337\epsilon)$ path-connected if any two points in $\mathbb{R}^2$ can be connected by a finite number of segments in $\mathbb{R}^2$.
3 are by a finite \( n \) of line segments in \( \mathbb{R} \), joined end to end.

Aside: Show:
If \( L_1, L_2 \) are open,
then so is \( R_1 \cap L_2 \).

pf: If \( \mathbb{R}_1 \cap \mathbb{R}_2 = \emptyset \) done
(\( \emptyset \) is open).
Otherwise: for any \( z \in \mathbb{R}_1 \cap \mathbb{R}_2 \):

- For \( \emptyset \) is open.
- Two points.
Since $\varepsilon_1 > 0$ (since $S_1$ is open),

$$S_1 \subseteq B_{\varepsilon_1}(z) \subseteq S_2.$$ 

Since $\varepsilon_2 > 0$ (since $S_2$ is open),

$$S_2 \subseteq B_{\varepsilon_2}(z) \subseteq S_2.$$ 

So $S = \min\{\varepsilon_1, \varepsilon_2\}$. Then

$$B_{\varepsilon}(z) \subseteq S,$$

by $\bigstar$ & $\bigstar$.

Since $\varepsilon$ was arbitrary,

$$S \cap \partial R_2,$$ we have shown that

$$\text{Int}(S \cap \partial R_2) = S \cap \partial R_2,$$ i.e.,

$R_1 \cap R_2$ is open. $\square$

* An open, connected subset $C$ is called a domain.

* A set whose interior is a domain is called a region.

* A pt $z \in C$ is an accumulation of a set $S \subseteq C$. 

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A set whose interior is a domain is called a region.

A pt $z \in \mathbb{C}$ is an accumulation pt of a set $S \subseteq \mathbb{C}$ if

every deleted nbhd of $z$ intersects $S$.

E.g. $0 \in \{ \frac{1}{n} \}_{n \in \mathbb{N}}$

\[ \frac{1}{2} \]

$0$ is only acc. pt.
E.g. \( S = B_1 \):
set of acc pts in \( \overline{B}_1 \).

\[ 815-816 \text{ Limits.} \]

Let \( f \) be defined on a deleted nbhd of \( z_0 \in C \),
with \( f \) taking values in \( C \).

\[ \lim_{z \to z_0} f(z) = w_0 \]

Given \( \varepsilon > 0 \) \( \exists \delta > 0 \) s.t.
\[ 0 < |z - z_0| < \delta \Rightarrow |f(z) - w_0| < \varepsilon \]

Note: \( f \) does \underline{NOT} have to be defined at \( z_0 \) for this to make sense.
E.g. \( \lim_{\theta \to 0} \frac{\sin \theta}{\theta} = 1 \)

Note: If the limit exists, it is unique.