§12 (8Ed §11) Basic

Topology

Given \( z_0 \in \mathbb{C} \) & \( \varepsilon > 0 \):
\[ B_\varepsilon (z_0) = \{ \text{open ball of radius } \varepsilon \text{ about } z_0 \}, \text{a.k.a. } \varepsilon\text{-neighbourhood} \]

\[ B_\varepsilon (z_0) = \{ z \in \mathbb{C} \mid |z - z_0| < \varepsilon \} \]

\[ \{ z \in \mathbb{C} \mid |z - z_0| \leq \varepsilon \} \]


\[ \overline{B}_\varepsilon (z_0) = \{ \text{closed ball of radius } \varepsilon \text{ about } z_0 \}, \text{a.k.a. closed } \varepsilon\text{-neighbourhood of } z_0 \]
\[ B_{\varepsilon}(z_0) = \{ z : |z - z_0| < \varepsilon \} \]

<table>
<thead>
<tr>
<th>In $\mathbb{R}$:</th>
<th>[ | (x+i\gamma) - (x_0+i\gamma) | ]</th>
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</thead>
<tbody>
<tr>
<td></td>
<td>$=</td>
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<tr>
<td></td>
<td>$= \sqrt{(x-x_0)^2 + (y-y_0)^2}$</td>
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<tr>
<td></td>
<td>$= | (x,y) - (x_0,y) |_2^2$</td>
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<tr>
<td></td>
<td>$= d((x,y),(x_0,y))_2^2 = d((z,z_0))_2^2$</td>
</tr>
</tbody>
</table>

radius

closed by

\[ B_{\varepsilon}(z_0) \]

\[ \overline{B}_{\varepsilon}(z_0) \]

\[ \varepsilon \text{-nbhd} \]

of \( z_0 \)
In $\mathbb{R}$:
\[
\begin{array}{c}
\delta > 0 \\
\frac{1}{2} \delta \\
\delta
\end{array}
\]

Take $z \in C$:
* $z \in C$ is an interior point of $\Omega$
\[
y \in \exists \delta > 0 \text{ st } B_{\delta}(z) \subset \Omega
\]
(note: $B_{\delta}(z) \subset \Omega$)
* $z \in C$ is an exterior point of $\Omega$
\[
m \in \exists \delta > 0 \text{ st } B_{\delta}(z) \cap \Omega = \emptyset
\]
$z \in \mathbb{C}$ is a boundary pt of $\Omega$.

$z \in \partial \Omega$ - bdy of $\Omega$, if $\forall \varepsilon > 0$

$B_\varepsilon(z) \cap \Omega \neq \emptyset$.

$B_\varepsilon(z) \cap \Omega^c \neq \emptyset$ ($\Omega^c = \mathbb{C} \setminus \Omega$ - complement of $\Omega$ (in $\mathbb{C}$)).

(i.e., any ball centered at $z$ hits both $\Omega$ & $\Omega^c$).

Note: interior pts belong to $\Omega$, exterior pts belong to $\Omega^c$; bdy pts may belong to either.
\*Int \( \mathcal{D} \) = interior of \( \mathcal{D} \) = \( \{ z : z \) is an interior \( \text{pt of} \mathcal{D} \} \)

\*Ext \( \mathcal{D} \) = exterior \( \mathcal{D} \) = \( \{ z : z \) is an exterior \( \text{pt of} \mathcal{D} \} \)

\*Bdy \( \mathcal{D} \) = \( \partial \mathcal{D} \) = boundary \( \text{pts of} \mathcal{D} \)

\( \mathcal{D} \) is open \( \iff \mathcal{D} = \text{Int} \mathcal{D} \)

\( \mathcal{D} \) is closed \( \iff \mathcal{D} = \text{Int} \mathcal{D} \cap \partial \mathcal{D} \)
$E \times 1 \quad \Omega_1 = B(0) = B_1 = B$

$\Omega_1 = \mathbb{D}$

$\text{Int } \Omega_1 = \mathbb{D}$

$\text{Ext } \Omega_1 = \{ z : |z| > 1 \}$

$\mathbb{S} \Omega_1 = \{ z : |z| = 1 \}$

$\Omega_1^c = \{ z : |z| \geq 1 \}$

$\Omega_1$ is open

$E \times 2 \quad \Omega_2 = B(0) = B_2 = B$

$\Omega_2 = \mathbb{D}$

$\text{Int } \Omega_2 = \mathbb{D}$

$\text{Ext } \Omega_2 = \text{Ext } \mathbb{D}$

$\mathbb{S} \Omega_2 = \mathbb{S} \mathbb{D} = \mathbb{D} \mathbb{S}$

$\Omega_2^c = \text{Ext } \mathbb{D}$

$\Omega_2$ is closed
Ex 3 \[ S_3 = \{ z : 0 < |z| < 1 \} \]

* \( \text{Int } S_3 = \{ z : 0 < |z| < 1 \} \)

* \( \text{Ext } S_3 = \text{Ext } \mathbb{R} \)

* \( \overline{S_3} = \text{Ext } S_3 \cup \{0\} \)

* \( \partial S_3 = S^1 \cup \{0\} \)

\( S_3 \) is neither open nor closed.

Note:

\( \mathbb{R}^1 \) open, \( \mathbb{R}^2 \) closed;

\( \mathbb{R}^3 = \mathbb{R}^2 \setminus \mathbb{R}^1 \)

\( \mathbb{R}^3 \) and \( \mathbb{R}^3^c \) neither open nor closed.

* \( \text{Int } \mathbb{R}^2 = \mathbb{R}^2 \)

* \( \text{Ext } \mathbb{R}^2 = \mathbb{R}^2 \)

* \( \partial \mathbb{R}^2 = \mathbb{R}^1 \)

* \( \mathbb{R}^2 \) is open

* \( \mathbb{R}^2 \) is closed