This is the in-class coding theory assignment for MATH3302, and contributes 3% towards your final assessment. This assignment is to be completed during the lecture and tutorial on Tuesday 5th May. Your completed answer page is due by the end of the tutorial (2:50pm). You will work in groups of two or three and submit one assignment per group. Marks will be given for in-class participation and for the submitted work.
Each person should receive a copy of the questions and each group should receive one copy of the answer page. Rough working should be done on scrap paper and final answers should be written in the appropriate places on the answer page. Please write your names and student numbers on the answer page and hand in only the answer page.

The following matrix is a generating matrix for a linear code C.

$$\mathbf{G} = \begin{pmatrix} 1 & 1 & 1 & 0 & 0 & 0 & 0 \\ 1 & 0 & 1 & 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 0 & 0 & 1 & 1 \end{pmatrix}$$

1. State the length and dimension of this code.

2. Find a generating matrix in RREF for the code C. Call this matrix $\mathbf{G}^*$.

3. Is C a systematic code? Explain your answer briefly.

4. **(a)** Use the generating matrix $\mathbf{G}^*$ to encode the message 101.

(b) Suppose that message words have been encoded using the generating matrix $\mathbf{G}^*$. Given a codeword, how would you determine the corresponding message word?

(c) Use the generating matrix $\mathbf{G}$ to encode the message 101.

(d) Suppose that message words have been encoded using the generating matrix $\mathbf{G}$. Given a codeword, how would you determine the corresponding message word?

5. Apply Algorithm 2.13 to find a parity check matrix for C.

6. **(a)** Use your parity check matrix for C to determine the distance of C and explain how you did this.

(b) How many errors can C detect and correct?

7. Let $\Theta_p(C)$ be the probability that if a codeword v of C is transmitted over a binary symmetric channel with reliability p, then the process of IMLD correctly concludes that v was sent. Without reference to the SDA which you will construct in question 8, explain why

$$\Theta_p(C) \geq p^7 + 7p^6(1-p).$$

8. Constructing an SDA for C by listing the cosets and finding the coset leaders would be a lengthy (and tiresome) process. Discuss with your group members how you could construct an SDA without listing the cosets of C. Then construct an SDA for C (without listing the cosets) and write a brief paragraph explaining how you did it.

continues overleaf

9. Now that you have an SDA for C, determine the actual value of $\Theta_p(C)$ in terms of p.

10. The generating matrix $\mathbf{G}^*$ was used to encode an English message using the following key.

space	A	E	I	O	D	L	N
000	100	010	001	110	101	011	111

The following string of bits has been received.

10001100111010111111001000001110111110001111110001100101

Use your SDA to determine the most likely intended English message. For each received word, indicate the most likely number of errors, and the most likely message word(s). Use the redundancy inherent in the English language to resolve any ambiguities and determine the most likely intended English message.

Name and Student Number:______________________________

Name and Student Number:______________________________

Name and Student Number:______________________________

MATH3302 **In-class Coding Theory Assignment – Answers**

Please write your answers in the appropriate places. If there is insufficient space, continue on another sheet of paper.

1. Length = Dimension =

2. $\mathbf{G}^* =$

3. YES NO (circle the appropriate answer)

____

____

____

____

4. **(a)** 101 encoded using $\mathbf{G}^*$ gives __________

(b) ____

____

____

____

(c) 101 encoded using $\mathbf{G}$ gives __________

(d) ____

____

____

____

5. $\mathbf{H} =$

6. **(a)** Distance of C is ____.

(b) C is ____-error detecting and ____-error correcting.

7.

8. How the SDA was constructed.

8. An SDA for C is given below.

coset leader(s)	syndrome

9. ______________________________

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10. The following table may assist you in the decoding of the message. If there is insufficient space, please continue on the back of this page.

received word	syndrome	number of errors	most likely message word(s)	corresponding English letter(s)
1000110				
0111010				
1111110				
0100000				
1110111				
1100011				
1111000				
1100101				

The most likely intended English message is: