Models of population growth under resource limitation discussed so far have considered a constant carrying capacity. In reality, of course, carrying capacity varies from year-to-year, because it depends on factors such as rainfall, frequency of frosts, and so forth. This year-to-year weather-driven variation does not undermine the basic principles of resource-limited population growth, although it may obscure the clarity of density-dependent trends.

A simple but useful extension of logistic population growth modelled with weather-driven variation has been applied to the Serengeti wildebeest populations. Wildebeests are regularly poached, and because their populations are currently quite large, Serengeti park officials would like to legalise a limited amount of harvesting. In allowing harvesting, park officials seek some compromise between maximising harvest and minimising the risk of wildebeest extinction (or herd collapse).

The problem is complicated by the fact that wildebeest demography is tightly connected to rainfall (because rainfall determines the amount of plant material produced each year as food), and rainfall varies unpredictably among years. A simple way of incorporating this complication is to define the wildebeests’ carrying capacity as a function of rainfall; in particular the relation $K = \text{rainfall} \times 5532$ has been shown to describe the dynamics of past wildebeest populations exceedingly well.

This $K$ is plugged into what is called the Beverton-Holt model for resource-limited growth. The Beverton-Holt model is one of a variety of different equations that qualitatively behave like the logistic equation, although their details vary. That is, at low population densities, the per-capita reproduction rate is the highest; and when carrying capacity is exceeded, the per-capita net reproduction rate falls to below 1 (so that the population shrinks). The Beverton-Holt equation for resource-limited growth is:

$$N_{t+1} = \frac{RN_t K}{K + (R - 1)N_t}$$

Where $R$ is the maximum geometric growth factor achieved as $N$ goes to zero and $K$ is the carrying capacity.

Q1. Using MATLAB, plot the Beverton-Holt equation showing population size vs time for a constant carrying capacity of $K=3,000,000$ and $R=1.1323$ over 99 iterations. Make two separate runs: one with initial population size of 250,000 and the other with 5,000,000 wildebeests. Explain what is happening.

Now let’s incorporate a variable carrying capacity and harvest rate so we can use MATLAB to investigate the management issue of harvesting in a variable environment. We now treat $K$ simply as a variable governed by the equation for rainfall, with rainfall drawn as a random variable from a list of past rainfalls recorded for the Serengeti. The idea then is to simulate future scenarios with different degrees of harvesting, where $h$ represents the fraction of population harvested. The complete model is thus:
\[ N_{t+1} = \frac{RN_tK}{K + (R - 1)N_t} \times (1 - h) \]

First create an array of actual observed rainfalls from the Serengeti,

\[ \text{rain} = [100, 38, 100, 104, 167, 167, 165, 79, 91, 77, 134, 192, 235, 159, \ldots, 211, 257, 204, 300, 187, 84, 99, 163, 97, 228, 208, 83, 44, 112, 191, \ldots, 202, 137, 150, 158, 20]; \]

and then generate the carrying capacities associated with each year of rainfall with

\[ K = 5532 \times \text{rain}; \]

Let’s consider a range of harvest rates,

\[ h = 0:0.02:0.2; \]

Using this information and the Beverton-Holt model you already have coded, write a Matlab programme that iterates wildebeest populations from one year to the next, according to the Beverton-Holt model to simulate the fate of wildebeest for each harvest level. Set the initial population size to be 250,000. Use an \( R \) of 1.1323 (which has been estimated from field data on population growth for wildebeests). Choose the rainfall level each year randomly, and its associated \( K \). Set a lower threshold of 100,000 wildebeests as a collapsed stock (this number is nearly the record low for the Serengeti herd).

Q2. Plot a graph of the wildebeest population through time under the different harvest rates. Explain what is happening. What harvest rate can the population sustain? Re-run your model again and explain why you have different a different answer.

Q3. Illegal hunting decreases the number of Serengeti lions, a major predator of wildebeest. How could you easily simulate predation with the model that you have? What effect does this have on the harvesting rate that is sustainable?

Q4. Much of Africa is forecast to have lower rainfall under future climate change scenarios. Assume that rainfall decreases to 30\% of its present value in the future. How does this affect the potential harvest rate (with and without illegal hunting)?

Q5. With a stochastic model, each model run will be different. Describe how you would extend your programme to confirm that the patterns that you’ve reported in this assignment hold and are not due to random variation?