MATH3104: Populations@Play

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Outline
• Investigate predator-prey interaction
  – without density dependence of prey
  – with density dependence of prey
  – with predator satiation and density
  dependence of prey
• Focus on cycles and equilibria

Predator-Prey Interaction
• Many big predators but most small
• Predators often liked more than prey???
• Questions
  – How many predators can prey sustain?
  – How much predation can the prey tolerate?
  – Are there cycles?

Volterra Predator-Prey Model
• 1st predator-prey model in ecology by Vito Volterra in 1920s
• Prey grow exponentially in absence of predator; predators die exponentially in
  absence of prey
• No density dependence
• Predators contact prey by “random collision”

Volterra Predator-Prey Model
\[
\frac{dN_1}{dt} = rN_1 - aN_1N_2 \\
\frac{dN_2}{dt} = baN_1N_2 - dN_2
\]

\[N_1\] Number of prey
\[N_2\] Number of predators
\[r\] Intrinsic rate of prey increase
\[a\] Coefficient relating prey capture to predator-prey collisions
\[b\] Coefficient relating prey capture to predator births
\[d\] Death rate of predator

Volterra Predator-Prey Model
• What does this model tell us?
• What are equilibria of these equations?
• What does your answer mean?
• Let’s plot system of equations and see what it looks like
Volterra Predator-Prey Model

- 5 main lessons
  1. Predators can control exponential prey growth
  2. Any prey population can support a predator population
  3. Formula for equilibrium abundances give estimate of average abundances through time
  4. Predator-prey interaction has tendency to oscillate
  5. The Volterra Principal: General pesticides ineffective controlling prey when usually controlled by predators

Adding Density Dependence to Prey

\[
\frac{dN_1}{dt} = rN_1 \left(1 - \frac{N_1}{K}\right) - aN_1N_2
\]

\[
\frac{dN_2}{dt} = bN_1N_2 - dN_2
\]

\[
\dot{N}_1 = \frac{d}{ba}
\]

\[
\dot{N}_2 = \frac{r}{a}(1 - \frac{d}{baK})
\]

How big does \( K \) of prey have to be to support the predator?

Volterra Predator-Prey with Density Dependence

- Let’s plot system and see what it looks like
- Density dependence has large effect on dynamics
- Value of \( K \) critical for coexistence

Volterra Model with Predator Satiation and Density Dependence

- Term for feeding is \( aN_1N_2 = (aN_1)N_2 \)
- Can use non-linear feeding term – e.g. \( c(1 - e^{-ct}) \)

Volterra Model with Predator Satiation and Density Dependence

\[
\frac{dN_1}{dt} = rN_1 \left(1 - \frac{N_1}{K}\right) - c(1 - e^{-\frac{a}{K}N_2})N_2
\]

\[
\frac{dN_2}{dt} = bc(1 - e^{-\frac{a}{K}N_2})N_2 - dN_2
\]

What does this system of equations look like?

\( K = 1000, \) Stable node \( K = 2000, \) Stable focus \( K = 4000, \) unstable focus surrounded by limit cycle