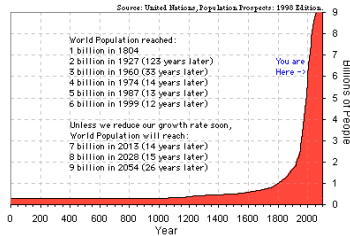


MATH3104

Lecture 4: Gotta stop somewhere



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Outline

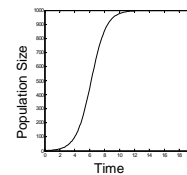
- Density dependence – intraspecific competition
- Including density dependence with geometric growth – the discrete logistic equation

Introduction

- Will *E. coli* cover world?
- One reason don't is consume resources, so per capita birth rate declines and death rate increases
- Population level off to steady-state size
- If average birth and/or death rate $f(\text{population size})$ then density dependent
- Are populations always limited this way?
- No - population bomb can be limited by natural fluctuations in environment (as we saw), without density dependence
- How to recognise density dependence and what models?
- Learn how many bald eagles or grizzly bears a particular habitat can support, or how many fish you can catch

Logistic Model of Limits to Growth

- Most elementary model of density dependence is logistic: S-shaped population curve



- 2 versions – discrete and continuous

Discrete Time Logistic

$$B(N) = B_0 - bN \quad \text{Note signs of } b \text{ and } d$$

$$D(N) = D_0 + dN$$

$B(N)$ = number of births

$D(N)$ = number of deaths

B_0 = density independent birth rate

D_0 = density independent death rate

b = coefficient relating individual birth to population size

d = coefficient relating individual death to population size

Geometric model $N_{t+1} = BN_t + (1 - D)N_t$

My 4.1 Let's derive the discrete logistic equation


Discrete Time Logistic...

$$\Delta N = rN \left(1 - \frac{N}{K}\right)$$


NOTE: r and K summary parameters lumping information on individual births and deaths

- N as stock or asset (natural capital)
- ΔN as interest (earnings) or growth (production) on capital (units number of organisms)
- ΔN is thus natural production function

Discrete Time Logistic...

M 4.1  Let's plot ΔN ...

Ex. 4.2  What are roots of the equation?


Ex. 4.3  Peak production is MSY – what is its equation?

- Ecologically optimal stock size, not economically optimal stock size (see later...)

Discrete Time Logistic...

- Logistic model also used to forecast population growth through time
- Simply iterate:

$$N_{t+1} = \Delta N + N_t$$

M 4.2  Let's plot logistic through time...