MATH 2400 Mid semester review: answers.

①

	$\liminf_{n\to\infty} a_n$	$\limsup_{n\to\infty} a_n$
a)	-1	1
b)	e^{-1}	e
c)	-4	4

② a) divergent (nth term test: e.g. show $a_{n+1}-a_n \geq 1 \ \forall n$, & hence $\lim_{n\to\infty} a_n = \infty$)
b) convergent (e.g. comparison with $\sum \frac{1}{2^n}$)
c) divergent (ratio test)
d) abs. convergent ("p-test", $p \geq 2$)
e) abs. convergent (comparison with geom series)
f) abs. convergent (ratio or root)

3a) Given $\varepsilon > 0$ want $\delta > 0$ s.t.

$$|x-2| < \delta \Rightarrow |f(x)-f(2)| < \varepsilon$$

$$\text{i.e. } |(x^2+7)-(2^2+7)| < \varepsilon$$

$$\text{i.e. } |x-2||x+2| < \varepsilon.$$

Choose $\delta < 1 \Rightarrow |x-2| < 1 \Rightarrow 1 < x < 3 \Rightarrow |x+2| < 5$.
So it's easy to check that $\delta < \min\{1, \varepsilon/5\}$ suffices.