MATH 2400 revision 2

① Use the integral test or other methods, determine whether the following series converge:

a) $\sum_{n=1}^{\infty} ne^{-n}$

b) $\sum_{n=1}^{\infty} \frac{1}{\sqrt{n+4}}$

c) $\sum_{n=3}^{\infty} \frac{n^2}{e^n}$

d) $\sum_{n=1}^{\infty} \frac{n}{n^5+1}$

② Find the derivative w.r.t. x of the following functions:

a) $\int_0^{\cos x} \cos(t^2)\,dt$

b) $\int_{2x}^{3x} \frac{u^2-1}{u^2+1}\,du$

③ Show that there holds

$$\sqrt{1+x} < 1 + \frac{x}{2} \quad \forall x > 0.$$

4 a) Show that $\tan x - 2x$ has a root in $(0, 1.4)$

b) Show that, for $f(x) = x^2 + 10\sin x$, there exists some $c \in \mathbb{R}$ with $f(c) = 1000$.