Math 2400
Assignment 4
Due 11:50 a.m. on 14 May, 2014

Drop your paper into the white box with your tutorial number on the fourth floor of the Priestley building

1. (10 points) For which $x \in \mathbb{R}$ does the series
$$\sum_{n=1}^{\infty} \frac{e^{-x^2}}{n^3 + x^2}$$
converge? Does it converge uniformly on $\mathbb{R}$?

2. (15 points) Define
$$f_n(x) = \frac{x^{2n}}{1 + x^{2n}}$$
for $x \geq 0$. Show that $(f_n)_{n=1}^{\infty}$ converges uniformly on every interval of the form $[0, a]$ with $a < 1$, and of the form $[b, \infty)$ with $b > 1$. What is the limit function in each case? Does $(f_n)_{n=1}^{\infty}$ converge uniformly on $[0, \infty)$? Justify your answer.

3. (15 points) Does there exist a continuously differentiable function $f : [1, 5] \to \mathbb{R}$ such that $f(1) < 0$, $f(5) > 3$ and $f'(x) \leq e^{-f(x)}$? Justify your answer.

4. (10 points) Suppose $f(x)$ equals x^2 when $x \in \mathbb{Q}$ and 0 when $x \notin \mathbb{Q}$. Prove that f is differentiable at 0 and find the derivative $f'(0)$.