Math 2400
Assignment 3
Due 11:50 a.m. on 30 April, 2014
Drop your paper into the white box with your tutorial number
on the fourth floor of the Priestley building

1. (a) (5 points) Is the function

    $$f(x) = e^{-x}$$

    uniformly continuous on $\mathbb{R}$? Justify your answer.

    (b) (5 points) Is the function

    $$f(x) = e^{-x^2}$$

    uniformly continuous on $[0, \infty)$? Justify your answer.

2. (10 points) Prove that the equation

   $$2 \left(\frac{2 + |x|}{1 + |x|} \right)^{1 + |x|} = 5$$

   has at least one solution on $\mathbb{R}$.

3. (a) (5 points) Assuming $f : \mathbb{R} \to \mathbb{R}$ is continuous, prove that $|f|$ is continuous.

    (b) (5 points) Provide an example of a function $f(x)$ on $\mathbb{R}$ such that $f(x)$ is discontinuous at every point of $\mathbb{R}$ but $|f(x)|$ is continuous on $\mathbb{R}$.

4. (10 points) Suppose $f : \mathbb{R} \to \mathbb{R}$ satisfies $f(x + y) = f(x) + f(y)$ for all $x, y \in \mathbb{R}$. Assume also f is continuous at 0. Prove that f is continuous on $\mathbb{R}$.

5. (a) (5 points) Suppose f and g are uniformly continuous and bounded on some interval I. Prove that the product function fg is uniformly continuous on I.

    (b) (5 points) Provide an example of two uniformly continuous functions (not necessarily bounded) whose product is NOT uniformly continuous.