MATH2400 Midsemester Revision

The following questions range from quite simple to very difficult. The midsemester itself will be somewhere between the two. Also note that the topics covered here are not an exhaustive list of potential exam topics. Students should also revise lectures, assignments and the other revision materials provided on the course website.

Question 1

Prove the following statements using an ϵ-N argument.

a) $\lim_{n\to\infty} \frac{2n+10}{n} = 2$
b) $\lim_{n\to\infty} \frac{\sin(n)}{n} = 0$
c) $\lim_{n\to\infty} \frac{\log(n)}{n} = 0$
d) $\lim_{x\to\infty} \frac{n^{100}}{n^{101}+n} = 0$
e) $\lim_{n\to\infty} 1 + \frac{(-1)^n}{n+1} = 1$
f) $\lim_{n\to\infty} n^{(1/n)} = 1$
g) $\lim_{n\to\infty} e^{-n} + \frac{1}{n} = 0$
h) $\lim_{x\to\infty} \frac{\sinh(n)}{e^n} = \frac{1}{2}$
i) $\lim_{n\to\infty} \frac{n^3+n^2-n}{4n^3+7} = \frac{1}{4}$
j) $\lim_{n\to\infty} \frac{\sqrt{n}}{n+1} = 0$
k) $\lim_{n\to\infty} e^{-n} + e^{1/n} = 1$
l) $\lim_{x\to\infty} \frac{2^n \sin(n)^n}{e^n} = 0$

Question 2

Prove the following statements using an ϵ-δ argument.

a) $\lim_{x\to 10} 2x + 10 = 30$
b) $\lim_{x\to -5/2} 1 - 4x = 11$
c) $\lim_{x\to 1} x^2 + 2x - 1 = 1$
d) $\lim_{x\to 0} \frac{x^2+2x-1}{5x^2+8} = \frac{1}{5}$
e) $\lim_{x\to 5} \frac{x^3+1}{x-x^3} = -\frac{21}{20}$
f) $\lim_{x\to 0} e^x = 1$
g) $\lim_{x\to\pi} -\sin(4x) = 0$
h) $\lim_{x\to -1} \sqrt{|x-5|} = \sqrt{6}$
i) $\lim_{x\to 0} \sqrt{x+1} - x = 1$
j) $\lim_{x\to 1} \log(5x^2) = \log(5)$
k) $\lim_{x\to 0} x^{1/3} + x = 0$
l) $\lim_{x\to\infty} \frac{x^2+1}{4x^2+x} = \frac{1}{4}$

Question 3

Determine whether the following series converge absolutely, converge conditionally, or diverge.

a) $\sum_{n=1}^{\infty} \frac{n^3+1}{n^4}$
b) $\sum_{n=1}^{\infty} \frac{1}{n^4+10}$
c) $\sum_{n=1}^{\infty} \frac{(-1)^n+n}{(-1)^n-n}$
d) $\sum_{n=1}^{\infty} \frac{3+\cos(n)}{e^n}$
e) $\sum_{n=1}^{\infty} \frac{2^n}{4^n}$
f) $\sum_{n=1}^{\infty} \frac{n!(n+1)!}{(3n)!}$
g) $\sum_{n=1}^{\infty} \frac{1+2^n}{3^{n-1}}$
h) $\sum_{n=1}^{\infty} \frac{(-1)^n}{\sqrt{n^2+1}}$
i) $\sum_{n=1}^{\infty} \frac{n}{2^n}$
j) $\sum_{n=1}^{\infty} (-1)^n \cos(1/n)$
k) $\sum_{n=1}^{\infty} \frac{n+5}{n\sqrt{n+3}}$
l) $\sum_{n=1}^{\infty} \frac{(-1)^n n!}{\pi^n}$

Question 4

Prove the following statements.

a) 2 is a cluster point of $\{(-1)^n + \cos(\pi n)\}_{n=1}^{\infty}$.
b) $\{-n + \sin(n)\}_{n=1}^{\infty}$ is decreasing.
c) $\{(2^n + 3^n)^{1/n}\}_{n=1}^{\infty}$ is increasing.
d) $\forall x, y \in \mathbb{R}$, $2xy \leq x^2 + y^2$.
e) $\lim_{x\to 0} \sin\left(\frac{1}{x}\right)$ does not exist.
f) $\forall \{a_n\}, \{b_n\}$, $|\sup_{n\in\mathbb{N}} a_n - \sup_{n\in\mathbb{N}} b_n| \leq \sup_{n\in\mathbb{N}} |a_n - b_n|$.
g) $\forall p \in \mathbb{Q}^+$, $\limsup_{x\to\infty} \sin(px)\sin(x) = 1$.