Second Semester 2008
MATH2200 Scientific Computing   Assignment 4 Ordinary Differential Equations

Weighting: 5% + optional 2% to replace Problem 6 of Assignment 3.
Due Date: 4.00pm on 12 Sept 2008, in the course Assignment Box in the Priestley Building (67).

Please hand in a hard-copy of the assignment (assignments on disk, CD, etc. will not be accepted). Include the code used to solve each problem. Comment your code extensively will help to understand your work. Uncommented code will lose marks.

1. (*2 points*) (*If you did not hand Problem 6 of the Assignment 3, or if you did not include this problem into the Assignment 3, you need to include this problem in the Assignment 4. Otherwise, go to Problem 2*). Use the approximation to the derivatives of Eq. 2 and Eq. 3 of the Assignment 3 to solve numerically the differential equation $y' = -y$ in the time interval $0 < t < 10$ with initial condition $y(0) = 1$. Compare the numerical solution to the exact solution $y^{exact}(t) = exp(-t)$ and calculate the discretization error ($E = y^{numeric} - y^{exact}$). Can you give an explanation to the erratic behavior of the numerical solution with Eq. 2?.

2. (*2 points*) The large-scale fisheries may led to a drastic reduction in the population of many fish species. Tuna is facing survival threats due to over fishing. Let's assume that y is the population of tuna fish (in mega-fishes), the growing rate of the fishes is one mega-fish per year, and the carry capacity of the ocean is one mega-fish per year. Let's suppose that the fishing rate is given by $0.1 + 0.01t$ mega-fish per year (where t is given in years). The model for population of fish can be represented by

$$y' = y(1 - y) - 0.1 - 0.01t, \tag{1}$$

Explain the meaning of each term of this equation. Solve numerically this equation with initial condition $y(0) = 0.25$ using trapezoid method. Plot the population of fishes versus time and use it to predict when the population of tuna fish crashes to zero. (hint: you can implement the trapezoid method by modifying the Program 6.1 of the Sauer's Book).

3. (*3 points*) Use the Euler and trapezoid methods to solve numerically the equation of the simple pendulum:

$$\frac{d^2\theta}{dt^2} + (g/l)sin(\theta) = 0. \tag{2}$$

Where $g = 10m/s^2$ is the gravity and $l = 0.1m$ is the length of the pendulum. Use a time step of $h = 0.001s$. Plot the angle of inclination of the pendulum with respect to the vertical θ versus the time using different initial conditions: $\theta(0) = 0, 0.1\pi, ..., 0.5\pi$ for both Euler and trapezoid methods. Finally plot the trajectories in the phase space ($d\theta/dt$ versus θ). You will observe two different phenomena: 1) the period of the oscillation of the pendulum depends on amplitude. 2)the trajectories on the phase space have a spiral shape. Are they real physical phenomena? or do they represent numerical artifacts?. (hints: Performing some physical experiments at home will help you to answer those questions. The numerical methods can be implemented by modifying Program 6.2 of the Sauer's Book)