MATH 2200 (Semester 2, 2008): Assignment 2

A/Prof Geoffrey Goodhill

This assignment is worth 10% of the total mark for the course overall. It is due by 4.00pm on Friday August 22nd, in the course Assignment Box in the Priestley Building (67).

This assignment is taken from computer problems in Sauer chapter 3. The chapter also contains some useful MATLAB programs for these problems which you may use and adapt. These programs are available on the computers in the computer lab.

You must hand in a hard-copy of the assignment for marking (assignments on disk, CD, etc. will not be accepted). Please comment your code extensively so it is obvious to the markers what you are trying to do. Uncommented code will lose marks. Note that MATLAB must be used to solve all problems and relevant code included in your answers. Please demonstrate the correct use of MATLAB functions to separate general algorithms from specific problems.

1. Apply the following world population figures to estimate the 1980 population, using (a) the straight line through 1970 and 1990 estimates; (b) the parabola through 1960, 1970, and 1990 estimates; and (c) the cubic curve through all four data points. Compare with the 1980 estimate of 4452584592. Hand in: MATLAB code for calculating estimates, comments on comparisons with the true estimate. (2 marks)

year	population
1960	3039585530
1970	3707475887
1990	5281653820
2000	6079603571

2. Write a MATLAB function polyinterp.m that takes as input a set of (x, y) interpolating points and another x_0, and outputs y_0, the value of the interpolating polynomial at x_0. The first line of the file should be function y0 = polyinterp(x,y,x0), where x and y are input vectors of data points. Use newtdd and nest and follow the structure of Program 3.2. Demonstrate your function works by applying to estimate the population size in 1980 using the data supplied in question 1. Hand in: MATLAB code for polyinterp.m, MATLAB session using polyinterp.m to estimate population in 1980. (3 marks)

3. (a) Use the addition formulas for sin and cos to prove that $\tan(\pi/2 - x) = 1/\tan x$. (b) Show that $[0, \pi/4]$ can be used as a fundamental domain for $\tan x$. (c) Design a function for interpolating a solution for the tangent, following the principles of Program 3.3, using degree 3 polynomial interpolation on this fundamental domain. (d) Empirically calculate the maximum error of the tangent key in $[0, \pi/4]$. Hand in: Proof for (a,b). MATLAB code for (c). MATLAB session for (d). (5 marks)

4. Build a MATLAB program to evaluate the cosine function correct to 10 decimal places using Chebyshev interpolation. Start by interpolating on a fundamental domain $[0, \pi/2]$, and extended your answers to inputs between -10^4 and 10^4. You may want to adapt code given in the chapter. Hand in: MATLAB code for cosine evaluation. MATLAB session showing evaluation at $\pi/4$, 3×10^3. (2 marks)

5. Find the clamped cubic spline that interpolates $f(x) = \ln x$ at five evenly spaced points in $[1, 3]$, including the endpoints. Empirically find the maximum interpolation error on $[1, 3]$. Plot the interpolation over the interval $[1, 3]$ along with the exact solution. Mark on the plots the positions of the 5 points used for interpolation. You may adapt code given in the chapter. Hand in: Relevant MATLAB code and MATLAB sessions used to create plots, plots. (5 marks)

6. (a) Consider the natural cubic spline through the world population data points in problem 1. Evaluate at the year 1980 and compare with the correct population. (b) Using a linear spline, estimate the slopes at 1960 and 2000, and use these slopes to find the clamped cubic spline through the data. Plot the spline and estimate the 1980 population. Which estimates better, natural or clamped? Hand in: Relevant MATLAB code and sessions for the splines and plots. Plot of the spline estimate. Comment on which gives an better estimate. (3 marks)