

## 4.1 Pattern formation

“Far less information is needed to specify a process of ‘becoming’ than to specify the resulting state of ‘being’!”

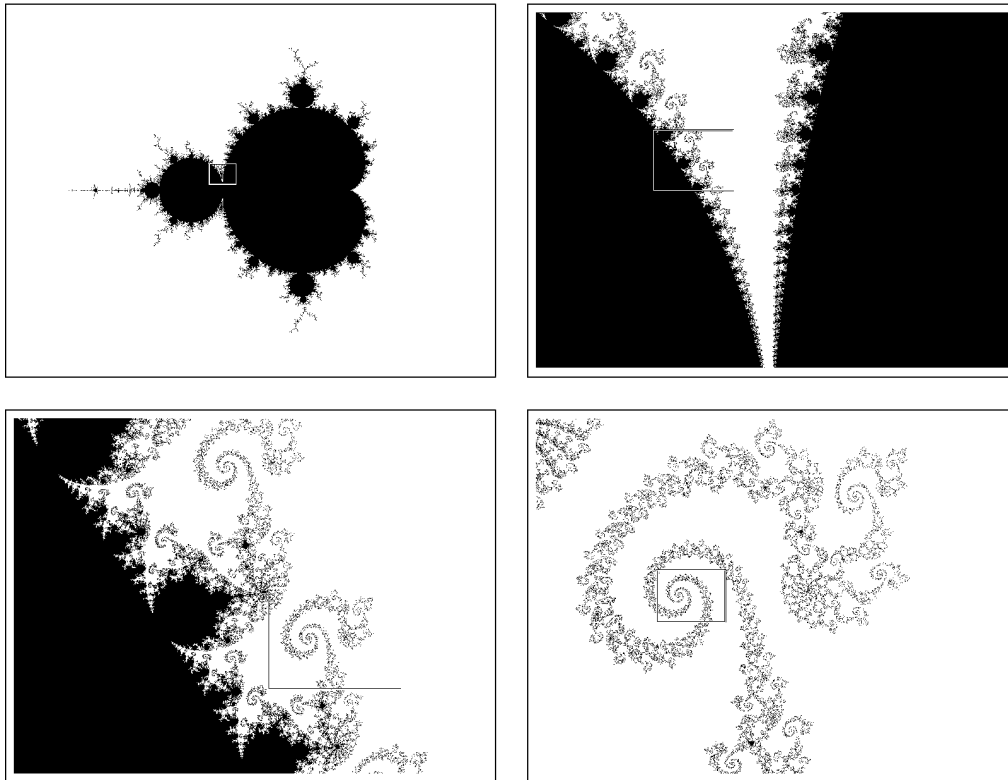
M.A. Corner p16, The Self-Organising Brain.

- The patterns we will consider in this lecture are repetitive spatial configurations. Examples from nature include the coat patterns of animals (*e.g.* leopards, fish, zebra and lizards), patterns on seashells and the branching patterns of coral, crystals, lightning and trees.
- See also the NetLogo models: Fur, Honeycomb, DLA, Segregation, Voting, Ising, BZ and the models in the Fractals and Crystallization folders.
- Some patterns might seem to be static but each pattern is generated by a dynamical system.

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## 4.2 Fractals and L-systems

- Mathematically, a **fractal** is a geometric pattern that is repeated at every scale (see Chapter 2 in CBoN for more on fractional dimensions, etc.).
- Examples of fractals:
  - Plants, brains, lungs, kidneys.
  - Coastlines and clouds.
  - Mathematical examples include bifurcation diagrams and the Mandelbrot set.
- **Self-similarity** refers to the way the structure looks the same when viewed at different scales and is synonymous with the word fractal.



- We will look at one way that fractal patterns can be generated: recursive algorithms, where a particular design is repeated on smaller scales indefinitely.
- **Recursion** simply refers to the way the process is repeated again and again on the same system.
- Koch curve:

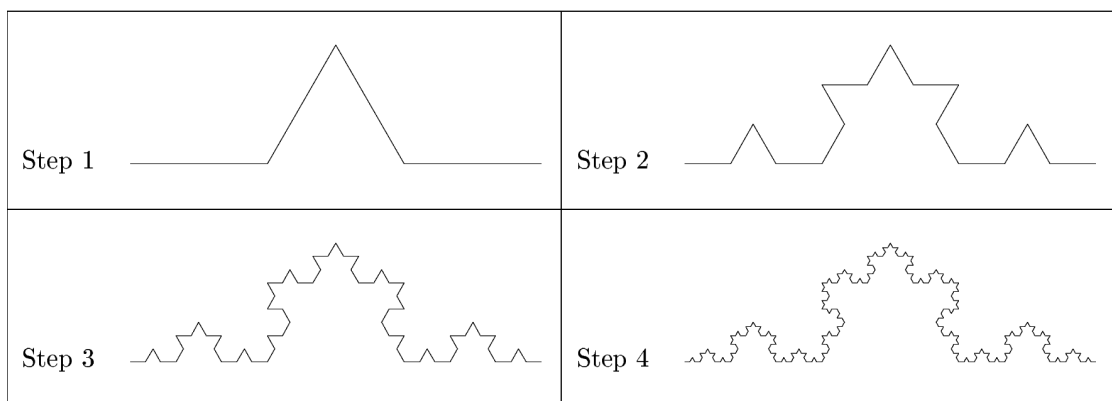
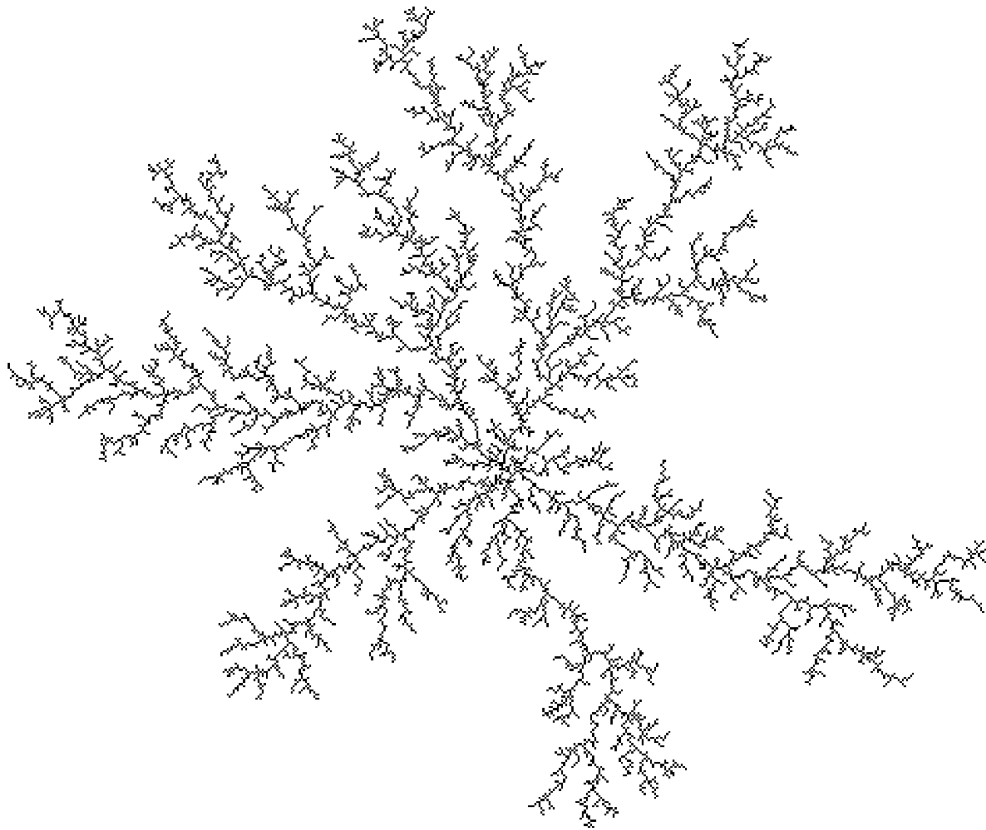


Figure 5.4 The first few steps in constructing the Koch curve

- L-systems:
    - rules implemented in parallel
    - start: A
    - rules:  $A \rightarrow B$ ;  $B \rightarrow AB$
    - ABBABBABBAB after 7 iterations.
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### 4.3 Diffusion Limited Aggregation

- Another method for generating branching structures is Diffusion Limited Aggregation.
- The NetLogo model demonstrates DLA in which randomly moving (diffusing) particles stick together (aggregate) to form branching fractal structures. There are many patterns found in nature that resemble the patterns produced by this model: crystals, coral, fungi, lightning, and so on.
- The model begins with an initial green "seed" in the center of the screen. Red particles move around the screen randomly. When a red particle hits a green square, it "sticks" and turns green (and a new red particle is created to keep the process going).
- This process is stochastic and generates a self-similar structure.
- It is important to understand how the different **ages** of the parts of the existing structure play a part in its final appearance.

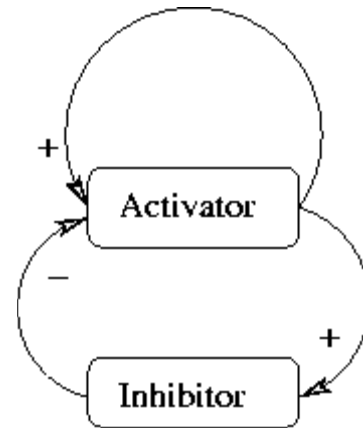


- What happens if the particles move around faster?
  - What happens if you start with more than one seed?
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#### **4.4 Reaction-Diffusion**

- Reaction-Diffusion systems demonstrate self-organisation and emergence in chemical systems.
- Around 1952, Alan Turing constructed a model of the possible mechanisms behind biological pattern formation (animal coats and seashells, etc.). He had the original idea that complex patterns could arise from simple interactions.
- A Reaction-Diffusion system consists of two chemicals: an activator and an inhibitor with local, fast reaction interactions and slower long-range diffusion.

- **Production of the activator** is inhibited by the presence of the inhibitor and enhanced by the presence of the activator (autocatalysis).
- **Production of the inhibitor** is also enhanced by the presence of the activator.



- To reach a stable state after some time:
  - Both substances are destroyed at a constant rate.
  - The inhibitor must diffuse faster than the activator.
  - The activator must enhance itself more than it enhances the inhibitor.
- Diffusion will try to equalise the concentrations over space while reaction changes the production rates locally: the whole system is affected by local change.
- The Belousov-Zhabotinsky reaction is a real Reaction-Diffusion chemical reaction that **doesn't move steadily towards an equilibrium state!** It is an example of recursive chemistry and oscillates back and forth between two states.  
 In a beaker, the colour changes from yellow to clear and back again, repeatedly.  
 But, in a thin layer of fluid, a dynamic pattern of spiralling coloured waves emerges.
- “For forty years after such systems were first described, most chemists did not believe they could work.” Figments of Reality, p16.
- R-D starts with random conditions and produces order!