Principles of Complex Systems

2.1 Characterising Complex Systems

- This first section will introduce many of the terms and concepts that describe properties of complex systems.
- **Complex systems** are made up of many interacting parts without any central authority. Complex systems can exhibit self-organisation and emergent behaviour.
- When each part of the system acts locally and autonomously, the system is called **decentralised**.
- **Self-organisation** is the ability of certain systems to create an ordered state in the absence of external pressures.

  "Self-organization is a process in which pattern at the global level of a system emerges solely from numerous interactions among the lower-level components of the system. Moreover, the rules specifying interactions among the system's components are executed using only local information, without reference to the global pattern."

  Camazine *et al.*, 2001, p8.

- **Emergence** can be described as coherent global behaviour arising from local interactions among lower level parts.

  Kauffman's terms 'antichaos' and 'order for free' both describe emergence.

- Emergent behaviour is **robust** since it is not particularly sensitive to initial conditions or perturbations. In systems that demonstrate emergence, order arises from many different initial states.
- Real world examples of emergence in complex systems:
  - Gene regulation
  - Organisms
  - Behaviour of insect colonies and swarms
  - Ecosystems
  - The economy
  - Cities
  - Intelligence
  - Evolution

Other definitions we need:

- A system is **stochastic** if it involves randomness. The opposite is deterministic. The randomness typically comes from our lack of knowledge of the underlying interactions or imprecise measurements. By one definition, randomness exists when repeated occurrences of the same phenomenon can result in different outcomes.
- **Feedback** is the process by which a system's behaviour is affected by the response it produces. Negative feedback decreases the system response and produces stable behaviour. Positive feedback amplifies the system response (a.k.a. the snowball effect).

2.2 Self-Organised Criticality

- This section introduces self-organised criticality (SOC) and the sand-pile model. We will look at the mechanisms of SOC and the characteristic behaviour. You should understand how a power law is represented mathematically and graphically.
- Self-organised criticality is a very general and far reaching phenomenon that unifies the understanding of many complex systems. It is the ability of a system to evolve in such a way as to approach a state where the system is barely stable and then maintain itself at that state.
- The archetypal example of SOC is a sand-pile maintaining a constant (critical) slope despite the rate of dropping sand or size of the initial pile.
- Self-organised criticality is characterised by a 'power law' distribution in some observable.
  In the case of sand-piles, the distribution of the sizes of sand slides or 'avalanches' follow the power law.
- The mathematical form of a power law is:

  $$N(s) \sim s^{-b},$$

  where N(s) is the number of events with size 's', and 'b' is the exponent.

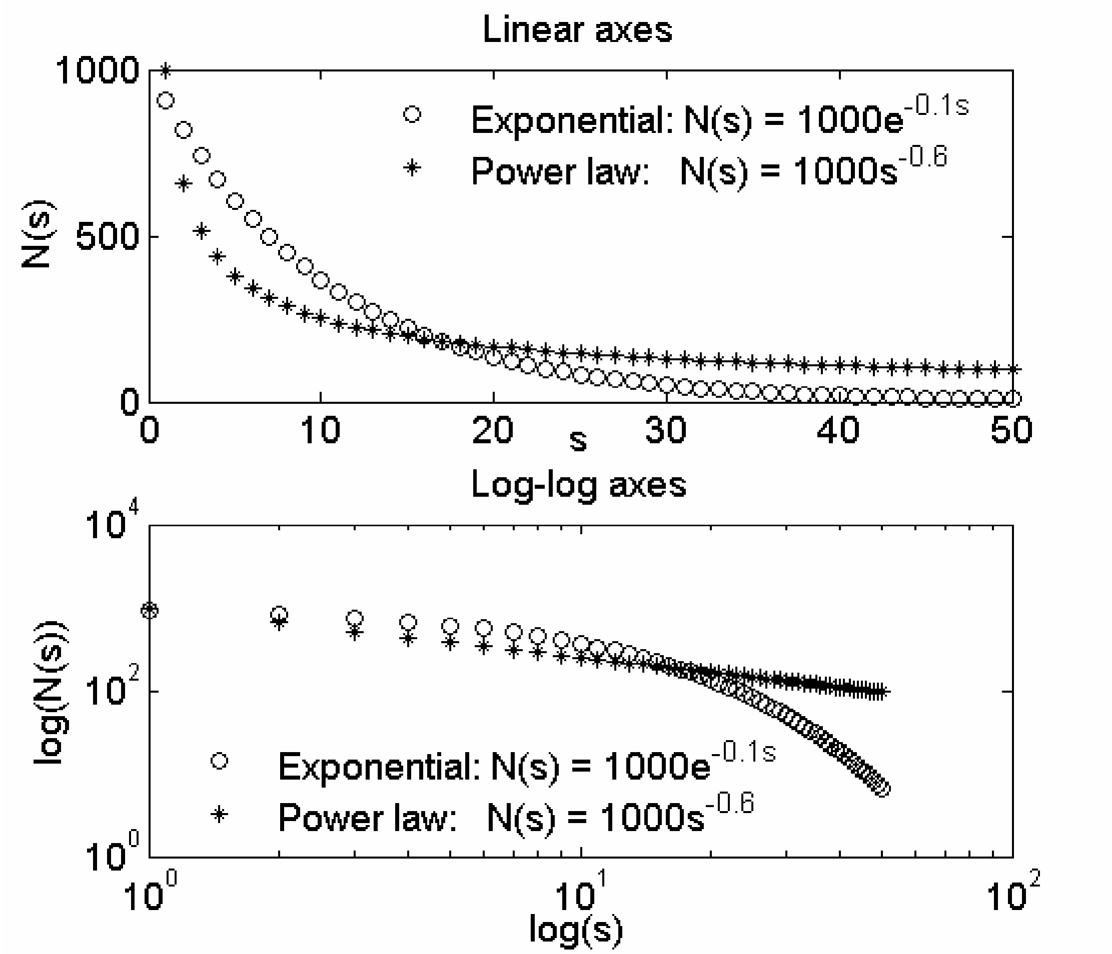
  Taking the log of both sides gives:

  $$\log N(s) \sim -b \log s,$$

  which appears as a straight line with gradient '$-b$' on a log-log plot.

o How does the coefficient $A\,s^{-b}$ show up on the log-log scale?

- The power law distribution is also called 'scale free' since it lacks a "characteristic length scale". It does not have a noticeable cut off like the exponential and Poisson distribution or a specified scale like the uniform distribution. The slope of the curve on any section of the log-log plot is the same.



- The point of all this is that power law behaviour is very common in nature:
  - Activity of the sun
  - Light from galaxies
  - Current flow in a resistor
  - Flow of water in a river
  - Earthquakes
  - Stock price fluctuations
  - Traffic flow in a city
  - Extinction of species
  - Signals in the brain
  - Even historical events
- Large pulses are rare, small ones are common but all sizes occur with a power law relationship in frequency.
- A steady trickle of sand or energy or water or social pressure drives the systems to organise themselves in the same way: a mass of interlocking parts just barely on the edge of criticality with breakdowns of all sizes ripping through and rearranging things just often enough to keep them on the edge.

2.3 Langton's Ant

- An example of a system with simple rules but unanticipated emergent behaviour is Langton's Ant:

  The ant is located on a grid whose cells can be either black or white. Initially the ant is started on a blank grid. At each time step the ant will be facing in one of four directions: N,S,E,W and follows a simple rule set at each time step:

  1. Take a step forward.
  2. If you are on a white cell, then paint it black and turn 90 degrees to the right.
  3. If you are on a black cell, then paint it white and turn 90 degrees to the left.

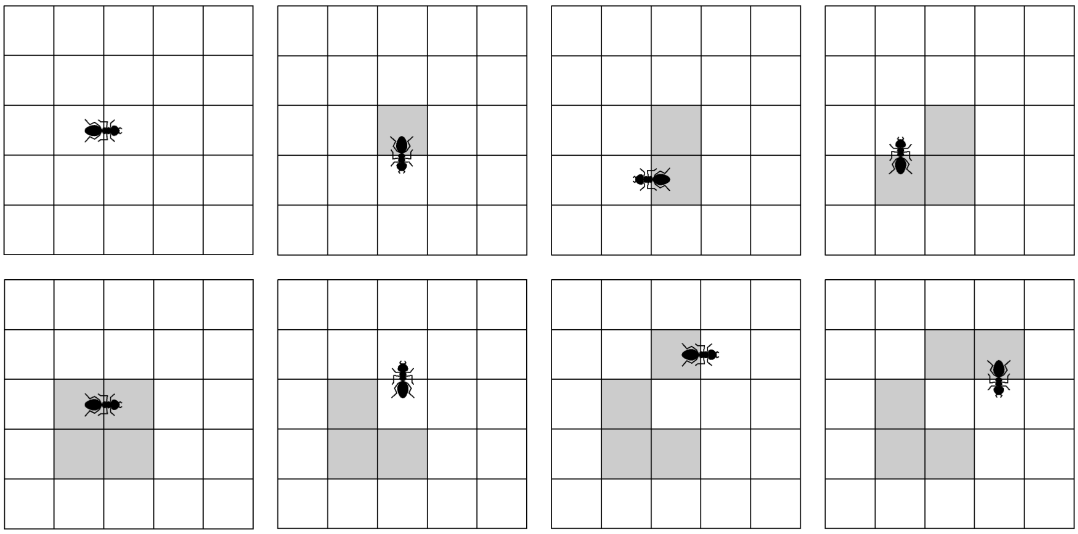


Figure 16.2 Eight steps of Langton's virtual ant, starting from an initially blank grid

- The behaviour of the system is deterministic and time reversible.
- What happens next?

- The ant wanders around for about 10,000 time steps forming a chaotic looking pattern with no apparent structure. However, after about another 250 steps the ant begins building a ‘highway’. It indefinitely generates a pattern of precisely 104 steps moving it two cells northwest, creating a diagonal band on the grid.

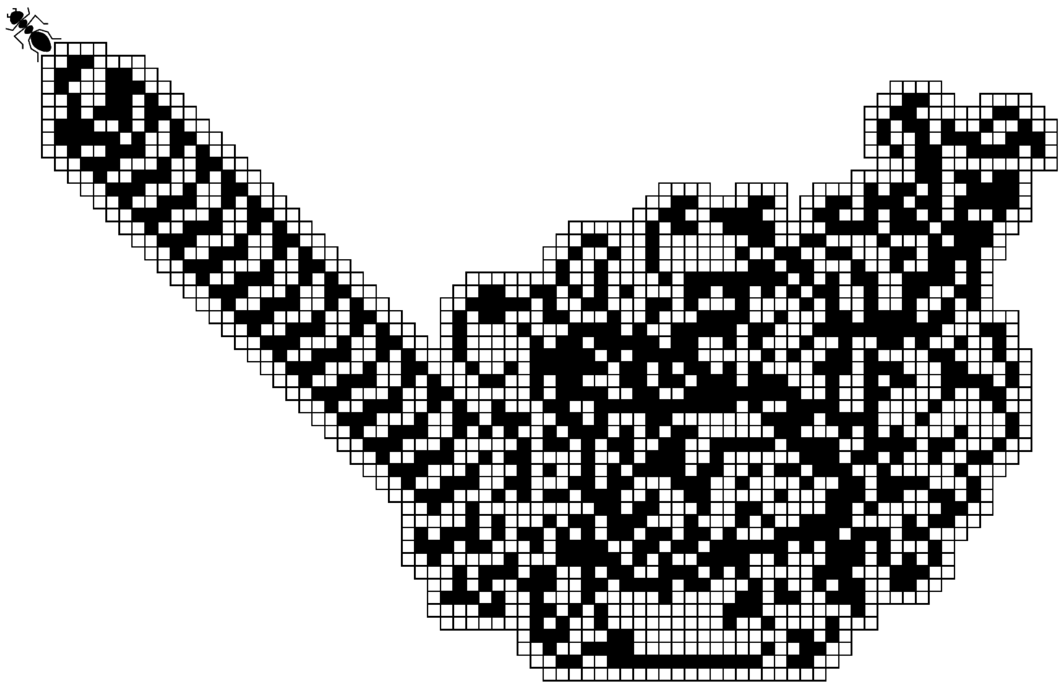


Figure 16.3 A virtual ant building a highway

- This highway epitomizes emergent behaviour: it is a large scale, highly ordered, unplanned, self-organized feature, unpredictable from knowledge of the rules, and robust to initial conditions.
- Even when agents obey simple rules it is not always possible to predict the future behaviour of such a system!

o How is this different to chaos?