MATH1070 Prac 1: Chaos

The Logistic Map

In this prac you will:

- Use MATLAB to iterate different initial conditions for the logistic map for different values of $r$.
- Draw trajectories in phase space using printed plots from MATLAB.
- Create a bifurcation diagram for the logistic map using nested loops in MATLAB.

Most of this work will be assessed in the first assignment.

Task 1: Dynamics

1. Write an m-file to iterate the logistic map: $x(t+1) = r \times (1-x(t))$. You should use a 'for' loop and may want to make it a function so that you can easily set the initial population size, the growth rate and the number of time steps. I suggest preallocating space for the variable $x$ with something like:
   
   ```matlab
   x=zeros(1,stop);
   ```
   
   This makes the program run faster and will be important later.

2. Pick a value of $r$ less than 2.8 and an initial $x$ in (0,1). Iterate the map until it ‘settles down’ and plot $x$ over time. Label the axes and add a title that includes the value of $r$ you chose.

3. Now choose a different initial condition in (0,1), iterate the map for the same number of time steps and add this new data to your plot with a different colour and/or marker (you will need to use the command `hold on`). Add a legend (easiest to use Insert → Legend on the figure then edit it) and label the data with the appropriate initial condition, something like $x(1)=0.23$. Describe what happens to the dynamics in response to the different initial conditions.

4. Create another figure as before, use $r=3.2$ and two different initial conditions of your choosing.

   Describe what happens to the dynamics in response to the different initial conditions this time.

5. Create another figure as before, use $r=3.9$ and choose two different initial conditions that are 0.0001 apart. Iterate the system for 60 time steps and describe what happens this time.
Task 2: Phase Space
1. Plot the line $y=r*x*(1-x)$ for $r=2.5$ and the line $y=x$ for $x$ in $(0,1)$. Turn the grid lines on and set both axes to show the interval $(0,1)$.
2. Print the plot.
3. Print out two more of the same plots, one for $r=3.2$ and one for $r=3.9$.
4. Pick any initial condition and use a pencil to trace the phase space trajectory and determine the behaviour of these maps.

Task 3: Bifurcation Diagram
To create a bifurcation diagram, we're going to use 4000 different values of $r$ and iterate the map 500 times for each value from the same initial condition. We will ignore the first 250 iterations as the burn in and plot the last 250 values as points for each value of $r$. The $x$ values will go on the y-axis and $r$ will be on the x-axis.

1. Write an m-file to create the data for the bifurcation diagram. I recommend using two nested for loops (one inside the other). After you set up the initial conditions and preallocate $x$ as a 4000 by 500 matrix, the outer for loop should increment $r$ by 0.001 from 0.001 to 4. The inner loop should iterate the map for 500 time steps with the current value of $r$.
2. Finally, you will need to plot the last 250 values of $x$ for each value of $r$.
   I recommend using small black dots ('k.', 'MarkerSize', 4).
3. Label the plot.
4. Plot the data again and zoom in: change the $r$ axis to go from 3 to 4.

Task 4: Analysis
1. Find the fixed point of the linear map: $x(t+1) = f(x(t)) = a + b \cdot x(t)$ and show that the period-2 values, $x(t+2) = x(t)$, are actually the same as the period-1 point.
   Hint, let $x(t+2) = x(t) = x^*$. Since $x(t+2) = f(x(t+1)) = f(f(x))$, solve $x^* = f(f(x^*))$.
   Under what conditions is the period-1 fixed point stable?
   Advanced: can you work out the closed form solution to the linear map?
2. Find both fixed points of the logistic map.
   For what values of $r$ do the two fixed points exist in $[0,1]$?
   How does the stability of each point change with $r$?
   Find the period-2 values.
   What equation do you have to solve to find the period three values?