Lecture 2 MATH1070 Christine Beveridge.

Reconsider the main steps of model development using box as example.

Go through the box example in L-studio showing the set-up of the program.

Show what this model teaches us.

Take example of yogurt maker. Can we come up with a model for making yogurt? The key information here is that the change in population over time (dP/dt) equals the sum of "flows in" minus the sum of "flows out".

Starting ingredients:

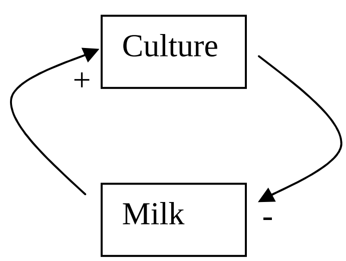
1. Yogurt culture (20 cells) and dried milk (10g)
2. 20 mL Water
3. Total Volume 30 mL

What is the relationship between the culture and the milk and the water?

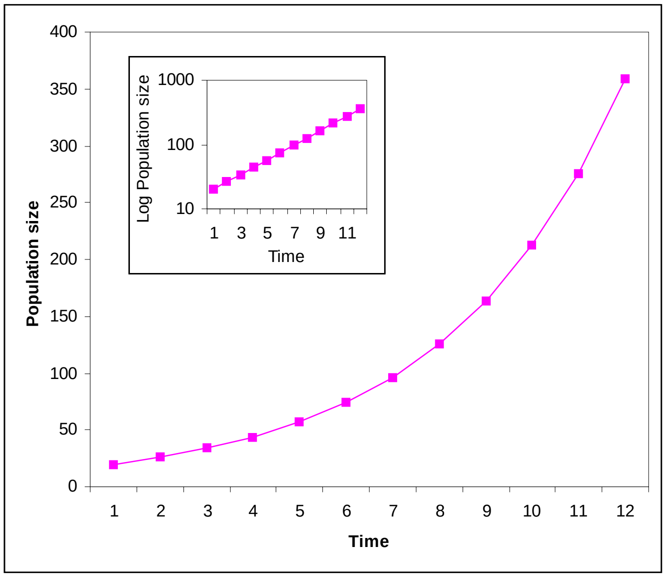
a. The culture cells consume the milk.
b. The milk is depleted by the culture.
c. The water and the volume is the environment.
d. There will be some limit or constraint provided by the container/water. The milk will eventually run out.
e. The culture may produce some waste which may be toxic.

Let us just focus on the milk and the culture (a and b, ignoring c to e for now).

Can we draw a diagram of this?



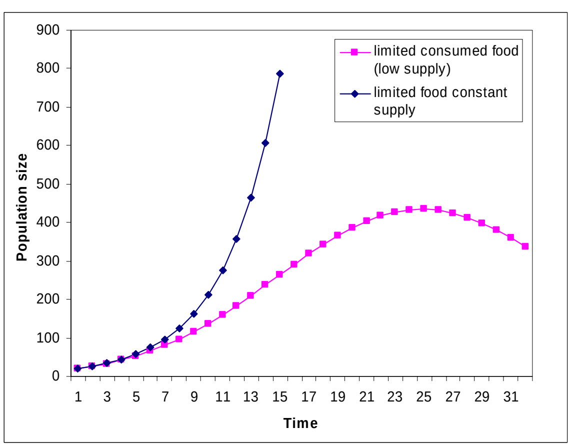
What is the quantity of culture at any point in time?
The culture is living so the change in culture from one moment to the next equals the number of births – the number of deaths. Assuming a homogenous population of cultures of different ages, we can assume that while there is plenty of food, the number of deaths is some constant x the number of cells. Similarly, the number of births will equal some constant times the x the number of cells. This leads to exponential growth.

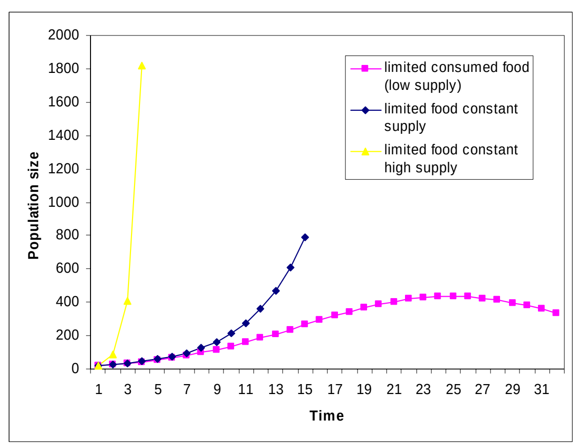


This exponential growth may occur while there is plenty of food and the environment is not limiting. But what if the food is limiting? What if the quantity of food affects the birth-rate, but not (directly) the death-rate? In this case the number of births would equal a new constant x the population size x the amount of food (milk). Of course, we need to also consider the food being depleted and that the change in food over time in this case is simply the amount of food consumed in that period.

We may state that that the food consumption is a constant *f* times the population size times the amount of food. If we consider a birth constant, *b*, in calculating the number of births then we have the number of births equals some constant, *b*, times *f* times the population size times the amount of food. In this way we recognize that in our initial description of the number of births, the constant actually captures the interplay of many constants.

Make sure you write down the differential equations that relate to this system.





On the left graph we see a nice response to a limited and dwindling food supply verses a limited constant food supply. On the right we show how growth is faster with a lot more food. Of course, organisms can only eat so much and at some point the quantity of food will not dictate growth and the growth constant will be the maximum growth rate.