1. Recall that the truth table for \( p \leftrightarrow q \) is

\[
\begin{array}{|c|c|c|}
\hline
p & q & p \leftrightarrow q \\
\hline
T & T & T \\
T & F & F \\
F & T & F \\
F & F & T \\
\hline
\end{array}
\]

Determine whether the following statement form is a tautology, a contradiction or neither.

\((\sim p \land \sim q) \leftrightarrow (p \lor q)\)

2. You need to design a circuit which takes three inputs, \( P, Q \) and \( R \), and has a single output \( x \) which should equal 1 if, and only if, all three inputs are equal.

(a) Create an input/output table corresponding to this specification.

\[
\begin{array}{|c|c|c|c|}
\hline
P & Q & R & x \\
\hline
1 & 1 & 1 & 1 \\
1 & 1 & 0 & 0 \\
1 & 0 & 1 & 0 \\
1 & 0 & 0 & 0 \\
0 & 1 & 1 & 1 \\
0 & 1 & 0 & 0 \\
0 & 0 & 1 & 0 \\
0 & 0 & 0 & 0 \\
\hline
\end{array}
\]
(b) Write down a logical expression for the output $x$.

(c) Draw a circuit corresponding to the logical expression for the output $x$.

3. Consider the following argument.

If wages are raised then buying increases. If there is a depression then wages are not raised and buying does not increase. Therefore either there is a depression or wages are raised.

(a) Let $w$ represent wages are raised, $b$ represent buying increases and $d$ represent there is a depression. Write the above argument in symbolic form.

(b) Use a truth table to determine whether the argument in Part (a) is valid or invalid.

<table>
<thead>
<tr>
<th>$w$</th>
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4. (a) Consider the following statement.

Given any integer $x$, $x^2$ is greater than 0.

(i) Write the statement in symbolic form.

(ii) Write the negation of the statement in symbolic form.

(iii) Which of the statement or its negation is true? Why?

(b) Consider the following statement.

There is a real number $x$ such that for every real number $y$, the product of $x$ and $y$ is 0.

(i) Write the statement in symbolic form.

(ii) Write the negation of the statement in symbolic form.

(iii) Which of the statement or its negation is true? Why?
(c) Let $P$ be the set of all people, and let $L(x, y)$ mean person $x$ loves person $y$. Consider the following statement.

$$(\forall x \in P)(\exists y \in P)(L(x, y) \land \sim L(y, x))$$

Write the statement in informal english (without using $\forall$, $\exists$, $\in$, $P$, $L$, $\land$, $\sim$).

5. Let $n \equiv 1 \pmod{3}$. Prove that $3 \mid (n^2 + 2)$. (Hint: if $n \equiv 1 \pmod{3}$ then $n = 3k + 1$ for some $k \in \mathbb{Z}$.)

6. Use a proof by contradiction to prove the following statement.

If $x$ is a non-zero rational number and $y$ is a (non-zero) irrational number then $x/y$ is irrational. (Note: $x/y$ means $x$ divided by $y$.)

7. Prove that each of the following two statements is false, by giving a counterexample to each of them.

(a) The product of any rational number and any irrational number is irrational.

(b) $\forall x, y \in \mathbb{R}, \lfloor x + y \rfloor = \lfloor x \rfloor + \lfloor y \rfloor$. 