Complete all of the following problems and hand in your solutions by the due date and time. Make sure that your name and student number are on each sheet of your answers. Solutions to all the problems will be distributed later. Late assignments will not be accepted unless you have a sufficient documented reason, such as illness.

1. For each of the following statements, determine whether it is true or false. Prove the statement directly from the relevant definitions if it is true, or give a counterexample if it is false.
   
   (a) The difference of the squares of any two consecutive integers is odd.
   (b) For all integers $n$, if $n$ is prime then $(-1)^n = -1$.
   (c) For all integers $m$, if $m > 2$ then $m^2 - 4$ is composite.
   (d) The difference of any two rational numbers is a rational number.
   (e) For all integers $a$, $b$ and $c$, if $a | bc$ then $a | b$ or $a | c$.
   (f) The sum of any five consecutive integers is divisible by 5.
   (g) For all integers $n$, if $n$ is odd then 
   \[ \left\lfloor \frac{n^2}{4} \right\rfloor = \frac{n^2 + 3}{4}. \]
   (h) For all real numbers $x$, $\lfloor x^2 \rfloor = \lfloor x \rfloor^2$.

2. Do each of the following:
   
   (a) If $n = -28$ and $d = 8$, find integers $q$ and $r$ such that $n = dq + r$ and $0 \leq r < d$.
   (b) Evaluate $39 \div 5$ and $39 \mod 5$.
   (c) When an integer $x$ is divided by 11, the remainder is 7. What is the remainder when $5x$ is divided by 11?
   (d) When an integer $x$ is divided by 5, the remainder is 4. What is the remainder when $6x$ is divided by 15?

3. Each of the following “proofs” is incorrect. Explain why each “proof” is wrong. (You are not required to prove the statements.)
   
   (a) The product of any four consecutive integers is divisible by 4.
   \[ \text{Proof: } \text{Consider the four consecutive integers } 1, 2, 3, 4. \text{ The product } 1 \cdot 2 \cdot 3 \cdot 4 = 24. \]
   Now 24 is divisible by 4. Hence product of four consecutive integers is divisible by 4.
   
   (b) For all integers $a$, $b$, $c$, if $a | bc$, then $a | b$.
   \[ \text{Proof: } \text{Let } a, b, c \text{ be integers and suppose that } a | b. \text{ Then } b = ra \text{ for some integer } r. \]
   Multiplying both sides of this equation by $c$ we get
   \[ bc = (ra)c = (rc)a. \]
   Since $rc$ is an integer, we know that $a | bc$. Hence if $a | bc$, then $a | b$.
   \[ \text{(continued over...)} \]
(c) The difference of any two odd integers is even.

**Proof:** Let \( m \) and \( n \) be odd integers. Suppose that \( m - n \) is even. Thus \( m - n = 2r \) for some integer \( r \). Now since \( m \) and \( n \) are both odd, we have

\[
m = 2s + 1 \text{ for some integer } s, \quad \text{and} \quad n = 2t + 1 \text{ for some integer } t.
\]

Thus

\[
m - n = (2s + 1) - (2t + 1) = 2r,
\]

so the difference of any two odd integers is even.

( the end )