EXERCISE 1·3·2

1.(a) page 15

Premise 1: $w \rightarrow b$

Premise 2: $d \rightarrow \sim w$

Conclusion: $\sim d \vee \sim w$

argument: $(w \rightarrow b) \wedge (d \rightarrow \sim w) \rightarrow (\sim d \vee \sim w)$

The argument is valid if this statement form is a tautology.

					A	B	C		
w	b	d	$\sim w$	$\sim d$	$w \rightarrow b$	$d \rightarrow \sim w$	$\sim d \vee \sim w$	$A \wedge B$	$A \wedge B \rightarrow C$
T	T	T	F	F	T	F	F	F	T
T	T	F	F	T	T	T	T	T	T
T	F	T	F	F	F	F	F	F	T
T	F	F	F	T	F	T	T	F	T
F	T	T	T	F	T	T	T	T	T
F	T	F	T	T	T	T	T	T	T
F	F	T	T	F	T	T	T	T	T
F	F	F	T	T	T	T	T	T	T

⇑

TAUTOLOGY ∴ ARGUMENT IS VALID

a shortcut...

					PREMISE	PREMISE	CONCLUSION	
w	b	d	$\sim w$	$\sim d$	$w \rightarrow b$	$d \rightarrow \sim w$	$\sim d \vee \sim w$	
T	T	T	F	F	T	F		
T	T	F	F	T	T	T	T	*
T	F	T	F	F	F			
T	F	F	F	T	F			
F	T	T	T	F	T	T	T	*
F	T	F	T	T	T	T	T	*
F	F	T	T	F	T	T	T	*
F	F	F	T	T	T	T	T	*

CRITICAL ROWS (WHERE ALL PREMISES ARE TRUE)

Since the conclusion is true in all rows where the premises are all true (in all "critical rows"), the argument is valid.

EXERCISES 1·3·2

1. (b) page 15

Premise 1: $c \rightarrow s$

Premise 2: s

Conclusion: c

$$(c \rightarrow s) \wedge s \rightarrow c$$

Argument is valid if this statement form is a tautology.

c	s	$c \rightarrow s$	$(c \rightarrow s) \wedge s$	$((c \rightarrow s) \wedge s) \rightarrow c$
T	T	T	T	T
T	F	F	F	T
F	T	T	T	F NOT A TAUTOLOGY
F	F	T	F	T

∴ ARGUMENT IS NOT VALID

The shortcut...

CONCLUSION	PREMISE	PREMISE	
c	s	$c \rightarrow s$	
T	T	T	* CRITICAL ROWS
T	F		
F	T	T	* CRITICAL ROWS
F	F		

In this critical row, the premises are all true but the conclusion is false. So the argument is INVALID.

EXERCISE 1·3·2
1.(c) page 15/16.

Premise 1: $c \vee j \to d$
Premise 2: $d \to r$
Premise 3: $\sim r$
Conclusion: $\sim c$

Argument: $(((c \vee j) \to d) \wedge (d \to r) \wedge \sim r) \to \sim c$

c	j	d	r	$\sim r$	$d \to r$	$c \vee j$	$c \vee j \to d$	$\sim c$	
T	T	T	T	F					
T	T	T	F	T	F				
T	T	F	T	F					
T	T	F	F	T	T	T	F		
T	F	T	T	F					
T	F	T	F	T	F				
T	F	F	T	F					
T	F	F	F	T	T	T	F		
F	T	T	T	F					
F	T	T	F	T	F				
F	T	F	T	F					
F	T	F	F	T	T	T	F		
F	F	T	T	F					
F	F	T	F	T	F				
F	F	F	T	F					
F	F	F	F	T	T	F	T	T	* ONLY CRITICAL ROW
				⇑	⇑		⇑		
				Premise 1	Premise 2		Premise 3		

In the critical rows the conclusion is true so the argument is valid.

SECTION 1·3: VALID ARGUMENTS

<u>EXTRA EXAMPLES</u> (not in workbook)

(1) If Peter does not have a real job then he does not wear shoes.
If Peter cannot tie his laces then he does not wear shoes.
Peter does not wear shoes.
Therefore Peter cannot tie his laces or he does not have a real job.

Let j represent "Peter has a real job",
s represent "Peter wears shoes", and
l represent "Peter can tie his laces".

The argument in symbolic form is:
Premise 1: $\sim j \rightarrow \sim s$
Premise 2: $\sim l \rightarrow \sim s$
Premise 3: $\sim s$
Conclusion: $\sim l \vee \sim j$.

Equivalently:
$$(\sim j \rightarrow \sim s) \wedge (\sim l \rightarrow \sim s) \wedge \sim s \longrightarrow \sim l \vee \sim j.$$

We search for a set of truth values which make the premises true but the conclusion false.
So $\sim j \rightarrow \sim s$ T
$\sim l \rightarrow \sim s$ T
$\sim s$ T
and $\sim l \vee \sim j$ F

But then s F (since $\sim s$ T)
$\sim l$ F and $\sim j$ F (since $\sim l \vee \sim j$ F)
so l T and j T
It is easy to check that j T l T s F make $\sim j \rightarrow \sim s$ T and $\sim l \rightarrow \sim s$ T also.

So j T, l T, s F is a set of truth values which make the premises all true but the conclusion false. So argument INVALID.

SECTION 1·3 VALID ARGUMENTS.

EXTRA EXAMPLES (cont.)

If Mario defeats Bowser, he rescues the Princess.
If Mario does not eat mushrooms then he doesn't defeat Bowser.
Mario does not rescue the Princess.
Therefore Mario does not defeat Bowser or does not eat his mushrooms.

Symbolically:

b: Mario defeats Bowser
p: Mario rescues Princess
m: Mario eats mushrooms

$$\begin{array}{l} b \to p \\ \sim m \to \sim b \\ \sim p \\ \therefore \ \sim b \vee \sim m \end{array}$$

$$(b \to p) \wedge (\sim m \to \sim b) \wedge \sim p \ \to \ \sim b \vee \sim m.$$

Suppose there is a set of truth values which make the premises true and the conclusion false.

So
$b \to p$ T
$\sim m \to \sim b$ T
$\sim p$ T
$\sim b \vee \sim m$ F

Then:
p F (since $\sim p$)
$\sim b$ F and $\sim m$ F (since $\sim b \vee \sim m$ F)
so b T and m T
But $b \to p$ T with b T and p F. NOT POSSIBLE!

We have a contradiction, and so our initial assumption (that there is a set of truth values which make the premises true but the conclusion false) is incorrect. So the argument is VALID.