Chap 1. Revision

① Definitions & Set Notation

\[ \mathbb{N}, \mathbb{Z}, \mathbb{Q}, \mathbb{Q}, \mathbb{R} \]

\[-3 \notin \mathbb{N}\]

② Trig

Solve \[2 \cos x = 1\] on \([0, 2\pi]\)

\[ x = \frac{\pi}{3}, \ 2\pi - \frac{\pi}{3} \]

③ Logs

Know

\[ \log(xy) = \log x + \log y \]

\[ \log x^a = a \log x \]

Solve

\[ 2 \ln x = \ln x \]

\[ \ln x^2 = \ln x \]

\[ x^2 = x \Rightarrow x = 0 \text{ or } 1 \]
Chapter 2 Functions

1. Domains & Ranges
   \[ y = x^2 + 5 \]
   \[ y = -\sqrt{x-3} \]
   \[ y = \frac{1}{x+2} + 3 \]
   Domain \( \mathbb{R} \) \( [3, \infty) \) \( \mathbb{R} \setminus \{ -2 \} \)
   Range \( [5, \infty) \) \( (-\infty, 0] \) \( \mathbb{R} \setminus \{ 3 \} \)

2. Graphs
   \[ y = ax^2 + bx + c \]
   \[ y = \frac{1}{x} \]
   \[ f(x) = -\sqrt{x-3} \]

[Graphs and diagrams omitted for clarity]
③ Inverse of a function

\[ y = f(x) = \frac{1}{x} + 3 \]
\[ y - 3 = \frac{1}{x} \quad x = \frac{1}{y - 3} \]
\[ f^{-1}(x) = \frac{1}{x - 3} \]

④ Compositions of functions

\[ h(x) = e^x \quad g(x) = x^2 \]
\[ g(h(x)) = (e^x)^2 \]

Need to be able to identify \( h \circ g \) given

\[ f(x) = \sin(x^2) \]
\[ h(x) = x^2 \]
\[ g(x) = \sin x \].
Chapter 3 Differentiation

1. Know the differentials of
\[ \frac{d}{dx} x^n = nx^{n-1}, \quad \frac{d}{dx} \sin x = \cos x, \quad \frac{d}{dx} \cos x = -\sin x, \]
\[ \frac{d}{dx} e^x = e^x, \quad \frac{d}{dx} \ln x = \frac{1}{x}. \]

2. Be able to use product, quotient & chain rule say to differentiate
   \[ x \sin x, \quad \frac{\sin x}{x}, \quad \sin 2x \]
   product, quotient, chain
   \[ u(x) = 2x \]

   \[ \frac{\sin x^2}{x^2}, \quad \frac{(x^2+5)^4}{x^2+5}, \quad \ln(x^3+2x) \]
   chain
   \[ u(x) = x^2, \quad u(x) = x^2+5, \quad u(x) = x^3+2x \]

3. Know the Defn of the derivative
\[ f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h} \]
use this to prove that
\[ f(x) = x^2, \quad \frac{1}{x}, \quad e^x, \quad (x+1)^2, \quad \frac{1}{x^2} \]
(4) Use derivative to calculate slope of a function at \( x = a \)

\[
\text{line} \quad y = f'(a)x + C \\
f(a) = f'(a) a + C \quad \text{solve for } C \\
(y - f(a) = f'(a)(x-a))
\]

(5) Sketch a complicated fn.

\[ y = x^3 - 2x^2 + x = f(x) \]

- **Intercepts**
  - Critical pts \( f'(x) = 0 \) \( x = a \)
  - 2nd deriv \( f''(x) > 0 \) min (local)
  - \( f''(x) < 0 \) local max
  - \( f'' > 0 \) slope +ve \( f'' < 0 \) slope -ve
  - \( f'' > 0 \) concave up \( f'' < 0 \) concave down

Use this in applications - be able to calculate global max/min

(6) Use Implicit Differentiation

\[
y^2 = x^3 + 4x \\
\frac{df(y(x))}{dx} = \frac{df}{dy} \frac{dy}{dx} \\
2y \frac{dy}{dx} = 3x^2 + 4
\]

\[
f(y) = y^2 \\
\frac{df}{dy} = 2y
\]
Chapter 4 Integration

1. Know integrals
\[ \int x^n \, dx = \frac{x^{n+1}}{n+1} + C, \quad n \neq -1 \]
\[ \int e^x \, dx = e^x + C \]
\[ \int \frac{1}{x} \, dx = \ln |x| + C \]

2. Use substitution to calculate integrals
\[ \int \frac{dx}{(2x+3)^2} \quad u(x) = 2x + 3 \quad \frac{du}{dx} = 2 \quad du = 2 \, dx \]
\[ \int 2x \sin x^2 \, dx \quad u(x) = x^2 \quad \frac{du}{dx} = 2x \quad du = 2x \, dx \]

3. Understand Riemann Sum
\[ \int_a^b f(x) - g(x) \, dx \]
\[ + \int_b^c g(x) - f(x) \, dx \]

4. Applications with distance, velocity, acceleration:
\[ v(t) = \int a(t) \, dt, \quad s(t) = \int v(t) \, dt \]
Reciprocal
\[
\frac{1}{f(x)}
\]

If \( f(a) = 0 \)
\[
\Rightarrow \frac{1}{f(a)} \rightarrow \infty
\]

\( f'(x) \)

\( \int f(x) \, dx \)

\( f(0) = 0 \)

where \( f(a) = 0 \) is a turning point for integral
Reciprocal

If \( f(x) < 0 \)
\[
\Rightarrow \frac{1}{f(x)} < 0
\]

If \( f(x) = 0 \) \( \Rightarrow x = a \) is a vertical asymptote for \( \frac{1}{f(x)} \)

Inverse
Reflect in \( y = x \) line

\( f'(x) \)
Which is the derivative? E
integral? B
reciprocal? C