1a) \[ \frac{x}{2} + 1 \leq 3 \]

\[ \Rightarrow -3 \leq \frac{x}{2} + 1 \leq 3 \]

subtract 1
\[ -4 \leq \frac{x}{2} \leq 2 \]

multiply by 2
\[ -8 \leq x \leq 4 \]

\( x \in [-8, 4] \)

b) \( \sqrt{x} \geq x \) \( \sqrt{x} \) only defined for \( x > 0 \)

\( y = x \)

From the diagram
\( \sqrt{x} > x \) for \( 0 \leq x \leq 1 \)

\( x \in [0, 1] \)

c) \[ \frac{2}{4 + x^2} \geq 1 \]

\[ \frac{2}{4 + x^2} \geq 1 \]

Since \( 4 + x^2 > 0 \) we can multiply through by it to give:
\[ 2 \geq 4 + x^2 \Rightarrow -2 \geq x^2 \] which is impossible

\[ \Rightarrow \text{No solutions} \quad \text{No number line} \]

2. \( h(x) = \sqrt{\frac{1}{x} + 2} = f \circ g(x) = f(g(x)) \)

where \( f(g) = \sqrt{g} \)

and \( g(x) = \frac{1}{x} + 2 \)

Now \( f(g) = \sqrt{g} \) has domain and range \([0, \infty)\)

\( g(x) = \frac{1}{x} + 2 \) domain \((-\infty, 0) \cup (0, \infty)\)

range \((-\infty, 2) \cup (2, \infty)\)

So \( h(x) \) is defined for \( x \neq 0 \) and \( \frac{1}{x} + 2 > 0 \)
Now \( \frac{1}{x} > -2 \)
if \( x \in (-\infty, -\frac{1}{2}) \cup (0, \infty) \)

So \( h(x) \in (-\infty, -\frac{1}{2}) \cup (0, \infty) \)

Now the range of \( \frac{1}{x+2} \) is \( \mathbb{R} \setminus \{2\} \)

So the range of \( h(x) \) is 
\( [0, \sqrt{2}) \cup (\sqrt{2}, \infty) \)

3. a) \( f(x) = \frac{1}{1+x^2} \)

is symmetrical about the y-axis. So \( x = a \) and \(-a\) give the same value. This means that the function is not 1-1.

However if we define \( f(x) \) on \([0, \infty)\)

this is a 1-1 function, with range \([0, 1]\)

Or if we define \( f(x) \) on \((-\infty, 0]\)

this is a 1-1 function with range \([0, 1]\)

Now the inverse of either of these functions is well defined.

For \( f(x) \) on \([0, \infty)\)
\( y = \frac{1}{1+x^2} \Rightarrow x = \sqrt{\frac{1}{y} - 1} \)

\( \Rightarrow f^{-1}(x) = \sqrt{\frac{1}{y} - 1} \) with domain \((0, 1]\)

For \( f(x) \) on \((-\infty, 0]\)
\( f^{-1}(x) = -\sqrt{\frac{1}{y} - 1} \) with domain \((0, 1]\)
4. a) The function \( \frac{x}{x+2} \) is well-defined at \( x = 2 \)

\[
\lim_{x \to 2} \frac{x}{x+2} = \frac{2}{2+2} = \frac{1}{2}
\]

b) Since both the top and bottom of this function are zero at \( x = 3 \), the limit is tricky, we must multiply by the conjugate:

\[
\lim_{x \to 3} \frac{x-3}{\sqrt{x+1} - 2} = \lim_{x \to 3} \frac{(x-3)(\sqrt{x+1} + 2)}{(\sqrt{x+1} - 2)(\sqrt{x+1} + 2)}
\]

\[
= \lim_{x \to 3} \frac{(x-3)(\sqrt{x+1} + 2)}{(x+1-4)} = \lim_{x \to 3} \frac{x-3}{\sqrt{x+1} + 2}
\]

\[
= \lim_{x \to 3} \frac{(x+1)^{-1} - 1}{x+1} = \frac{1}{x+1} - 1 = \frac{1 - (x+1)}{x+1} = -\frac{x}{x+1}
\]

So \( \frac{(x+1)^{-1} - 1}{x} = -\frac{x}{x(x+1)} = -\frac{1}{x+1} \)

and the limit becomes

\[
\lim_{x \to 0} \frac{-1}{x+1} = -1
\]
5 \quad f(x) = \frac{2}{x} \implies f(x+h) = \frac{2}{x+h}.

f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h} = \lim_{h \to 0} \left( \frac{2}{x+h} - \frac{2}{x} \right)

Now \quad \frac{2}{x+h} - \frac{2}{x} = \frac{2x - 2(x+h)}{x(x+h)} = -\frac{2h}{x(x+h)}

f'(x) = \lim_{h \to 0} -\frac{2h}{x(x+h)}

= \lim_{h \to 0} -\frac{2}{x(x+h)} = -\frac{2}{x^2}

6.a) Graphing the function on the left and right of the inequality and noting that

\[ y = \ln x, \quad \frac{dy}{dx} = \frac{1}{x} \]

So the slope at \( x = 1 \) is 1.

Then we can see that \( x-1 = \ln x \)

for \( x = 1 \) but \( x-1 \) is never greater than \( \ln x \).

So \( \ln x \leq x - 1 \)

for all values of \( x \) (for which \( \ln x \)

b) The slope of \( y = e^x \), \( \frac{dy}{dx} = e^x \),

at \( x = 1 \) is 1.

For \( x \neq 1 \), \( e^x > 1 + x \)

So there are no solutions.