MATH1050/7050 Semester 1, 2011

Week 8 Tutorial Problems

Work through the following problems, show your tutor then record your name before the end of your Week 8 tutorial. You are encouraged to discuss these questions and your solutions with your peers and to ask your tutor for assistance. Working through ten sets of tutorial problems is compulsory and each of the ten problem sets will contribute 0.5% towards your final grade.

Note that you earn the 0.5% for your effort in solving these problems during the tutorial rather than for answering all the problems correctly. Once you have finished these problems, you can use the remainder of your tutorial time to work on other aspects of the course. Solutions to the tutorial problems will be available at the end of the week.

1. Let \( F(x) = \int e^{2x} \, dx \), with \( F(0) = 1 \). Find \( F(x) \).

2. A rocket takes off vertically at time \( t = 0 \) with acceleration \( a(t) = 4t + 2 \). When \( t = 3 \) the rocket has velocity \( v(3) = 28 \), and when \( t = 3 \) the rocket has displacement \( S(3) = 40 \). Find expressions for the rocket's velocity and displacement at any time \( t \).

3. Find the equation of the function \( f(x) \) given that it passes through \((1, 5)\) and is such that \( f'(x) = ax - \frac{b}{x^2} \), where \( f(2) = 10 \) and \( f'(-1) = 2 \).

4. Let \( f(x) = x^2 - 9x + 8 \).
   a) Evaluate \( \int_0^9 f(x) \, dx \)
   b) Determine the physical area enclosed by \( f(x) \) and the \( x \)-axis from \( x = 0 \) to \( x = 9 \). Why is this answer different to a)?
   c) What does a) tells us about the graph of \( f(x) \)?

5. Write the following 2-space vectors in component and matrix form:
   a) vector \( \mathbf{v} \) has magnitude 2 and direction \( \frac{\pi}{6} \).
   b) vector \( \mathbf{w} \) has magnitude 4 and direction \( \frac{\pi}{3} \).

6. Find the magnitude and direction (in both degrees and radians) of the following vectors: a) \( \mathbf{v} = -\mathbf{i} - \sqrt{3}\mathbf{j} \) \hspace{1cm} b) \( \mathbf{w} = -4\mathbf{i} + 2\mathbf{j} \)

(continued over...)
7. a) Let \( \mathbf{a} = \begin{pmatrix} 3 \\ -2 \\ 5 \end{pmatrix} \) and \( \mathbf{b} = \begin{pmatrix} -2 \\ 4 \\ 8 \end{pmatrix} \). Determine the vectors \( \mathbf{c} \) and \( \mathbf{d} \) in matrix form, where

\[
\mathbf{c} = 3\mathbf{a} + \frac{1}{2}\mathbf{b} \quad \text{and} \quad \mathbf{d} = -2\mathbf{b} - \mathbf{a}.
\]

b) Let \( \mathbf{u} \) and \( \mathbf{v} \) be the vectors illustrated below.

Use geometric vector addition and scalar multiplication to illustrate the vectors

\[
\mathbf{c} = 2\mathbf{u} + \mathbf{v} \quad \text{and} \quad \mathbf{d} = \mathbf{v} - \mathbf{u}.
\]

8. Let \( \mathbf{a} = \begin{pmatrix} 3 \\ -2 \\ 5 \end{pmatrix} \).

a) Determine the norm of \( \mathbf{a} \).

b) Determine the matrix form of the unit vector \( \hat{\mathbf{a}} \).

9. Let \( \mathcal{ABCD} \) be a rectangle with \( P \) the midpoint of the line segment \( \mathcal{AC} \) as shown. Let \( \overrightarrow{AD} = \mathbf{u} \) and let \( \overrightarrow{AB} = \mathbf{v} \).

a) Express \( \overrightarrow{AP} \) in terms of \( \mathbf{u} \) and \( \mathbf{v} \).

b) Express \( \overrightarrow{BP} \) in terms of \( \mathbf{u} \) and \( \mathbf{v} \).

c) Express \( \overrightarrow{PD} \) in terms of \( \mathbf{u} \) and \( \mathbf{v} \).

d) Use the above calculations to show that the diagonals of a rectangle bisect each other.