1. a) Yes. Common difference, \( d \), of 4. A closed form is \( 4n - 1 \), \( n = 1, 2, 3, 4 \). A recursive definition is \( a_n = a_{n-1} + 4 \), \( a_0 = 3 \), \( n = 1, 2, 3 \).

b) Yes. Common difference, \( d \), of 2x. A closed form is \( (2n - 11)x \), \( n = 1, 2, 3, 4 \). A recursive definition is \( a_n = a_{n-1} + 2x \), \( a_0 = -9x \), \( n = 1, 2, 3 \).

c) No. No common difference.

2. a) Yes. Common ratio, \( r \), of \( \frac{1}{2y} \). A closed form is \( \frac{(-1)^{n-1}}{2^n \cdot y^n - 1} \), \( n = 1, 2, 3 \). A recursive definition is \( a_n = \frac{1}{2y}a_{n-1} \), \( a_0 = \frac{1}{2} \), \( n = 1, 2 \).

b) No. No common ratio.

c) Yes. Common ratio, \( r \), of \( x + 1 \). Closed form is \( (x + 1)^{n-1} \), \( n = 1, 2, 3 \). A recursive definition is \( a_n = (x + 1)a_{n-1} \), \( a_0 = 1 \), \( n = 1, 2 \).

3. \( a = -7, d = 4 \)

\[
S_n = \frac{1}{2} (2a + (n - 1)d)
\]

\[
1025 = \frac{1}{2} (-14 + (n - 1)4)
\]

\[
2050 = n(-14 + 4n - 4)
\]

\[
2050 = -14n + 4n^2 - 4n
\]

\[
4n^2 - 18n - 2050 = 0
\]

\[\Rightarrow n = 25 \text{ or } -20.5 \text{ (not possible).} \]

Therefore 25 terms are needed for the series to sum to 1025.

4. A.S. \( 6, 11, 16, ... \)

\( a_n = a + (n - 1)d \), where \( a = 6, d = 5 \)

So \( a_9 = 6 + 8 \times 5 = 46 \) and \( a_{16} = 6 + 15 \times 5 = 81 \)

5. G.S. \( 3, 4, 5\frac{1}{3}, 7\frac{1}{9}, ... \)

\( a = 3, r = \frac{4}{3} \)

\[
S_n = a \left( \frac{r^n - 1}{r - 1} \right)
\]

\[
500 = \frac{3((\frac{4}{3})^n - 1)}{\frac{4}{3} - 1}
\]

\[
(\frac{4}{3})^n - 1 = 55.5...
\]

\[
(\frac{4}{3})^n = 56.5...
\]

\[\Rightarrow n = \frac{\ln 56.5...}{\ln \frac{4}{3}} \approx 14.03, \text{ so 15 terms will be required for the series to exceed 500.} \]

6. \[\text{man of 100 packages = 10.4 tonnes}\]

\[
2 \times 2 = 4 \text{ kg}
\]

\[
S_n = \frac{n}{2} (2a + (n-1)d)
\]

\[
16400 = \frac{100}{2} (2a + 99 \cdot 2)
\]

\[
208 = 2a + 198
\]

\[
\Rightarrow a = 5
\]

\[\text{The heaviest package is 5kg, the heaviest 203 kg.}\]
7. \[ P_n = P_0 (1 + k)^n \]
\[ P_8 = 2700 (1 + 0.024)^8 \]
\[ = 2700 \cdot 1.024^8 \]
\[ \approx 3393.8 \ldots \]
\[ \approx 3394 \text{ tonnes} \]

Not particularly accurate given weather conditions can change very quickly. Long period of time (8 years) at 2.9% increase may not be an accurate representation.

\[ P_n = P_0 (1 + k)^n \]
\[ n = \frac{\ln \left( \frac{P_n}{P_0} \right)}{\ln (1 + k)} \approx 6.635 \ldots \]
\[ \approx 7 \text{ years (or something during the 7th year)} \]

9. a)

We need to show this statement is true for \( n = 1 \), that is, \( \sum_{j=1}^{1} 2^{j-1} = 2^1 - 1 \).

The left-hand side gives \( \sum_{j=1}^{1} 2^{j-1} = 2^0 = 1 \) and the right-hand side gives \( 2^1 - 1 = 1 \). Thus the statement is true for \( n = 1 \).

Now suppose that the statement is true for \( n = k \), so \( \sum_{j=1}^{k} 2^{j-1} = 2^k - 1 \).

We need to show that it is true for \( n = k + 1 \), that is \( \sum_{j=1}^{k+1} 2^{j-1} = 2^{k+1} - 1 \).

\[
\begin{align*}
\sum_{j=1}^{k+1} 2^{j-1} &= 2^0 + 2^1 + 2^2 + \ldots + 2^{k-1} + 2^k \\
&= \sum_{j=1}^{k} 2^{j-1} + 2^k \\
&= 2^k - 1 + 2^k \text{ since it is true for } n = k \\
&= 2^{k+1} - 1 \\
&= 2^{k+1} - 1
\end{align*}
\]

Therefore, if the statement is true for \( n = k \), then it is true for \( n = k + 1 \). Hence, by mathematical induction \( \sum_{j=1}^{n} 2^{j-1} = 2^n - 1 \) for all integers \( n \geq 1 \).
96) Prove \( n(n+1)(n+2) \) is divisible by 6, \( n \geq 1 \)

So is true when \( n = 1 \)

Assume \( P(k) = 6M \), where \( M \) is an integer

\[ P(k+1) = (k+1)(k+2)(k+3) \]
\[ = k(k+1)(k+2) + 3(k+1)(k+2) \]
\[ = 6M + 3(k+1)(k+2) \quad \text{(inductive assumption)} \]

But \( (k+1)(k+2) \) is even because the product of any two consecutive nos. is even.

So we now have

\[ 6M + 3 \times 2N \quad \text{where } N \text{ is an integer} \]

\[ = 6M + 6N \]
\[ = 6(M+N) \]

\[ \therefore P(k+1) \] is divisible by 6

\[ \therefore n(n+1)(n+2) \] is divisible by 6

for all \( n \geq 1 \) integers
9c) Prove \( 1 + 3 + \cdots + (2n-1) = n^2 \)

\( n=1 \) 
\[ \begin{align*}
\text{LHS} &= 1 \\
\text{RHS} &= 1^2 \\
\therefore \text{True when } n &= 1
\end{align*} \]

Assume \( S_k = P(k) \)
\[ \begin{align*}
\text{LHS} &= 1+3+5+\ldots+(2k-1) = k^2 \\
P(k+1) &= (k+1)^2 \\
&= k^2 + 2k + 1
\end{align*} \]

\[ \begin{align*}
S_{k+1} &= S_k + P(k+1) - 1 \\
&= P(k) + 2(k+1) - 1 \\
&= k^2 + 2k + 2 - 1 \\
&= k^2 + 2k + 1 \\
&= P(k+1) \\
\therefore S_n &= P(n) \\
1+3+5+\ldots+(2k-1) &= n^2 \]