Work through the following problems, show your tutor then record your name before the end of your Week 12 tutorial. You are encouraged to discuss these questions and your solutions with your peers and to ask your tutor for assistance. Working through ten sets of tutorial problems is compulsory and each of the ten problem sets will contribute 0.5% towards your final grade. Note that you earn the 0.5% for your effort in solving these problems during the tutorial rather than for answering all the problems correctly. Once you have finished these problems, you can use the remainder of your tutorial time to work on other aspects of the course. Solutions to the tutorial problems will be available at the end of the week.

1. Which of the following are arithmetic sequences? Give reasons. Find both a closed form and recursive definition for the arithmetic sequences.
   a) 3, 7, 11, 15
   b) \(-9x, -7x, -5x, -3x\)
   c) \(4y, 6y^2, 8y^3\)

2. Which of the following are geometric sequences? Give reasons. Find both a closed form and recursive definition for the geometric sequences.
   a) \(\frac{1}{2}, -\frac{1}{4y}, \frac{1}{8y^2}\)
   b) \(-4, -2, -1, -\frac{1}{2}, 0\)
   c) \(1, x + 1, x^2 + 2x + 1\)

3. How many terms of the A.S. \(-7, -3, 1, 5, \ldots\) must be taken for the corresponding series to sum to 1025?

4. Find the 9th and 16th terms of the A.S. 6, 11, 16, ...

5. How many terms of the G.S. 3, 4, 5\(\frac{1}{3}\), 7\(\frac{1}{5}\), ... must be taken for the corresponding series to exceed 500?

6. The total mass of 100 packages is 10.4 tonnes. If the masses of consecutive packages differ by 2kg, find the masses of the heaviest and lightest packages.

7. The annual grain harvest on a farm on the Darling Downs in 2001 was 2700 tonnes. Because of favourable conditions, the harvest yield increases by 2.9% each year. What is the expected harvest in 2009? Comment on the accuracy of your answer.
8. The Brisbane City Council has decided that all old-style see-saws are to be replaced. If there were 200 see-saws in 1995, and they are being replaced at a rate of 55% per year, how long will it be until there is only one old-style see-saw left?

9. Use mathematical induction to prove the following:

a) \( \sum_{j=1}^{n} 2^{j-1} = 2^n - 1 \), for all integers \( n \geq 1 \).

b) \( n(n + 1)(n + 2) \) is divisible by 6, for all integers \( n \geq 1 \).

c) \( 1 + 3 + 5 + \ldots + (2n - 1) = n^2 \)