# Math1050 Mathematical Foundations

**St Lucia Campus**  
**Semester One 2010**

**Final Examination**

**Mathematical Foundations**

- **Perusal Time:** 10 mins. During perusal, write on the blank paper provided
- **Writing Time:** 2:00 Hours
- **Examiner:** Mr Michael Jennings & Dr Barbara Maenhaut

This examination paper has 19 pages (not including title page and attachments) and is printed Single-Sided.

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**Exam Type:** Closed Book - Specified materials permitted

**Permitted Materials:**
- Calculator - Yes - EAIT/EPSA approved (must have label)
- Dictionary - Yes - Unmarked paper Bilingual Dictionary only
- Other -
  - No electronic aids are permitted (e.g. laptops, phones)

**Answer:** (Where students should write answers) All Questions on Examination paper in spaces provided

**Other Instructions:** You may detach the formulae sheet at the back.

**Total Number of Questions:** (for the whole examination) 25

**Total Number of Marks:** 80 total marks

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Students must comply with the General Award Rules 1A.5 and 1A.7 which outline the responsibilities of students during an examination.
PART A: For each of the following thirteen multiple choice questions, enter the letter corresponding to the correct answer in the corresponding box. There is no need to show any working. Each correct answer is worth 1 mark; each incorrect answer is worth 0 marks.

1. \[
\begin{vmatrix}
2 & 0 & -2 \\
0 & 2 & 2 \\
2 & 4 & 0 \\
\end{vmatrix} = \\
\]
(A) 8  
(B) 10  
(C) -10  
(D) -24  
(E) -8  
(F) 16  

Answer to Question 1: 

2. \[
\left( \begin{array}{cc}
2 & 0 \\
0 & -3 \\
\end{array} \right)^4 = \\
\]
(A) \[
\left( \begin{array}{cc}
8 & 0 \\
0 & -12 \\
\end{array} \right) \\
\]
(B) \[
\left( \begin{array}{cc}
8 & 0 \\
0 & -27 \\
\end{array} \right) \\
\]
(C) \[
\left( \begin{array}{cc}
8 & 0 \\
0 & 81 \\
\end{array} \right) \\
\]
(D) \[
\left( \begin{array}{cc}
16 & 0 \\
0 & -81 \\
\end{array} \right) \\
\]
(E) \[
\left( \begin{array}{cc}
16 & 0 \\
0 & 81 \\
\end{array} \right) \\
\]
(F) \[
\left( \begin{array}{cc}
16 & 0 \\
0 & 27 \\
\end{array} \right) \\
\]

Answer to Question 2: 

3. Determine \[
\lim_{x \to 2} \frac{3x^2 - 6x}{x - 2}.
\]
(A) -3  
(B) 0  
(C) 3  
(D) 6  
(E) 18  
(F) Does not exist

Answer to Question 3: 

4. If \( y = x^2e^x \), which one of the following statements is true?

(A) \( \frac{dy}{dx} = x^2e^x + 2xe^x \)

(B) \( \frac{dy}{dx} = 2xe^x + x^2e^{x-1} \)

(C) \( \frac{dy}{dx} = 2xe^x \)

(D) \( \frac{dy}{dx} = x^2e^{x-1} \)

(E) \( \frac{dy}{dx} = x^2e^x \)

(F) \( \frac{dy}{dx} = 2x^3e^x \)

Answer to Question 4: 

5. Which one of the following is the solution of the inequality \( 2 + | - x | > 4 \)?

(A) \([-2, 2]\)

(B) \((-2, 2]\)

(C) \((-\infty, -2) \cup (2, \infty)\)

(D) \((-\infty, -2] \cup [2, \infty)\)

(E) \((-\infty, -2) \cup [2, \infty)\)

(F) \((-\infty, -2] \cup (2, \infty)\)

Answer to Question 5: 

6. If \( y = 3x^2 - 3 + 2\sin x \) which one of the following statements is true?

(A) \( \int y \, dx = x^3 - 3x + 2\sin x + C \)

(B) \( \int y \, dx = x^3 - 3x + 2\cos x + C \)

(C) \( \int y \, dx = x^3 - 3x + \sin^2 x + C \)

(D) \( \int y \, dx = x^3 - 3x - 2\sin x + C \)

(E) \( \int y \, dx = x^3 - 3x + 2\cos^2 x + C \)

(F) \( \int y \, dx = x^3 - 3x - 2\cos x + C \)

Answer to Question 6: 

7. Let \( f(x) = \sqrt{x - 1} \). Which one of the following is the domain of \( f \circ f \circ f \)?

(A) \([0, \infty)\)

(B) \([1, \infty)\)

(C) \([2, \infty)\)

(D) \([3, \infty)\)

(E) \([4, \infty)\)

(F) \([5, \infty)\)

Answer to Question 7:   

8. If \(3y^2 = 4x^3 - 7\), which one of the following statements is true?

(A) \(\frac{dy}{dx} = \frac{12x^2 - 7}{6y}\)

(B) \(\frac{dy}{dx} = \frac{2x^2 - 7}{y}\)

(C) \(\frac{dy}{dx} = \frac{2x^2}{y}\)

(D) \(\frac{dy}{dx} = 12x^2 - 6y\)

(E) \(\frac{dy}{dx} = \frac{y}{2x^2}\)

(F) \(\frac{dy}{dx} = \frac{\sqrt{4x^3 - 7}}{3}\)

Answer to Question 8:   

9. For which value of \(x\) are the vectors \((x, -4)\) and \((3, 6)\) perpendicular?

(A) \(x = -2\)

(B) \(x = 8\)

(C) \(x = -8\)

(D) \(x = 2\)

(E) \(x = 21\)

(F) \(x = -21\)

Answer to Question 9:   

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10. For which value of $x$ are the vectors $(-4, x)$ and $(2, -3)$ parallel?

(A) $x = -6$

(B) $x = 9$

(C) $x = 3$

(D) $x = -3$

(E) $x = -9$

(F) $x = 6$

Answer to Question 10: [Blank]

11. If $S = (x^2 + 1) + (x^4 + 4) + (x^6 + 9)$, which one of the following statements is true?

(A) $S = \sum_{i=1}^{3} (x^{2i} + i)$

(B) $S = \sum_{i=1}^{3} (x^{2i} + i^2)$

(C) $S = \sum_{i=1}^{3} (x^{3i-1} + i^2)$

(D) $S = \sum_{i=2}^{4} (x^i + i^2)$

(E) $S = \sum_{i=2}^{4} (x^{2i-2} + 2i)$

(F) $S = \sum_{i=2}^{4} (x^{ixi} + i^2)$

Answer to Question 11: [Blank]
12. The graph of the function \( f \) with rule \( y = f(x) \) is shown below.

Which one of the following could be the graph of an antiderivative function of \( f \)?

(A) \hspace{2cm} (B) 

(C) \hspace{2cm} (D)

(E) 

Answer to Question 12: 

13. For the function $f(x)$ shown in the box, which one of the following would be the best representation of $\frac{1}{f(x)}$?

Answer to Question 13: [Blank]
PART B: SHOW ALL WORKING

14. Given matrices \( A = \begin{pmatrix} 1 & 2 \\ -3 & 0 \\ 2 & 4 \end{pmatrix} \) and \( B = \begin{pmatrix} 0 & 1 & -3 \\ -5 & 3 & 2 \end{pmatrix} \), evaluate the following where possible. If not possible, give a brief explanation why.

(a) \( A + 2B \)  

(b) \( A^T + B \)  

(c) \( AB \)  

(d) \( AB^T \)
15. Let $\mathbf{a} = i - 2j$ and $\mathbf{b} = 4i + 3j$. Complete the following:

(a) Determine $3\mathbf{a} - 2\mathbf{b}$. (1 mark)

(b) Determine $||\mathbf{a}||$. (1 mark)

(c) Determine $\mathbf{b}$. (1 mark)
16. Let $c = 5i - j$ and $d = 2i + 3j + k$. Complete the following:

(a) Evaluate $c \cdot d$.  

(b) Determine $c \times d$.  

(c) Determine the angle (in degrees) between $c$ and $d$. Give your answer to two decimal place accuracy.
17. Sketch the graph of the curve \( y = -x^4 + 2x^2 - 1 \) on the interval \([-2, 2]\), indicating on your graph any global/local maxima/minima as well as any intercepts and asymptotes. Show all working. (6 marks)
18. (a) Find values for $m$ and $n$ in the following arithmetic progression

$$7 - 2\sqrt{2}, m, n, \sqrt{2} - 2.$$  

(3 marks)

(b) Find values for $a$ and $b$ in the following geometric progression

$$2\frac{2}{3}, a, b, 1\frac{1}{8}.$$ 

(3 marks)
19. Use Mathematical Induction to show that \[ \sum_{i=1}^{n-1} i(i+1) = \frac{n(n-1)(n+1)}{3}, \]
for integers \( n \geq 2. \) (6 marks)
20. Determine the following integrals.

(a) \( \int \frac{4 + u^2}{u^3} du \)  

(b) \( \int_0^2 x^2 \sqrt{1 + x^3} \, dx \)
21. Consider the complex number $w = 3 - i$.

(a) Write down the real and imaginary parts of $w$. 

(b) Determine $|w|$ and $\text{Arg}(w)$. 

(c) Let $z = \frac{3 + i}{3 - i}$. Write $z$ in Cartesian form.
22. In outer space, object A, with a mass of 4 kg and moving at 1.0 m/s, collides with object B, with a mass of 6 kg and moving at 3.0 m/s, moving as shown in diagram (a). After the collision, object A moves off as shown in diagram (b). Calculate the speed and direction of object B after the collision. (7 marks)
23. (a) Sketch a graph of the function $y = |x + 2|$. Show $x-$ and $y-$intercepts. (2 marks)

(b) Evaluate $\int_{-3}^{3} |x + 2| \, dx$. (3 marks)
24. A group of ten people go to Suncorp Stadium to watch the Wallabies play Ireland, and each buys one drink during the game. Admission costs $49 for people 16 years and over and $25 for people aged under 16. The people who are 18 years old or over each buy a beer, those who are aged 16 or 17 buy a coke and those who are under 16 buy orange juice. The drink costs are $6 for beer, $4 for coke, and $5 for juice. Together they pay $370 for admission and $53 for drinks. Use matrices to determine how many people buy each type of drink.

Note: you can do this question without needing to find the inverse of a $3 \times 3$ matrix.

(6 marks)
25. \( P(x_1, y_1) \) is a point on the parabola \( y = x^2 \). If \( O \) is the origin, show that the area bounded by \( OP \) and the curve is one-sixth of the area bounded by the axes and the lines \( x = x_1, y = y_1 \). 

(5 marks)
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Cosine Rule: \( a^2 = b^2 + c^2 - 2bc \cos A \)

Sine Rule: \( \frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c} \)

\[
\sin(\theta + \phi) = \sin \theta \cos \phi + \cos \theta \sin \phi
\]

\[
\sin(\theta - \phi) = \sin \theta \cos \phi - \cos \theta \sin \phi
\]

\[
\cos(\theta + \phi) = \cos \theta \cos \phi - \sin \theta \sin \phi
\]

\[
\cos(\theta - \phi) = \cos \theta \cos \phi + \sin \theta \sin \phi
\]

\[
\cos 2\theta = \cos^2 \theta - \sin^2 \theta = 2 \cos^2 \theta - 1 = 1 - 2 \sin^2 \theta
\]

\[
\sin 2\theta = 2 \sin \theta \cos \theta
\]

For an arithmetic sequence of the form \( \{a_j\}_{j=1}^n \), where \( a_j = a + (j - 1)d \),
\[
S_n = \sum_{i=1}^{n} a_i = \frac{n}{2}(2a + (n - 1)d).
\]

For a geometric sequence of the form \( \{a_j\}_{j=1}^n \), where \( a_j = ar^{j-1} \),
\[
S_n = \sum_{i=1}^{n} a_i = \frac{a(1 - r^n)}{1 - r} = \frac{a(r^n - 1)}{r - 1}.
\]