INTERNAL STUDENTS ONLY

THE UNIVERSITY OF QUEENSLAND

Second Semester Examination, November, 2005

MATH 1050
Mathematical Foundations
(Unit Courses)

Time: TWO Hours for working
Ten minutes for perusal before examination begins

CREDIT WILL BE GIVEN ONLY FOR WORK WRITTEN ON THIS EXAMINATION SCRIPT.
Use the back pages if the space provided is insufficient, or for rough working.
Questions carry the number of marks shown. The total number of marks is 60.
Attempt all questions.
Pocket calculators allowed.

FAMILY NAME (PRINT): ________________

GIVEN NAMES (PRINT): ____________________________

STUDENT NUMBER: _______________________

SIGNATURE: ____________________________

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TOTAL

EXAMINER'S USE ONLY

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TURN OVER
1) (i) The vector $v$ has magnitude 4 and direction $\frac{\pi}{3}$. The vector $w$ has magnitude 6 and direction $\frac{\pi}{6}$. Find the projection of $v$ onto $w$. (5 marks)
1) (ii) Calculate the area of the triangle OAB where A is the point with coordinates (1, 1, 1) and B is the point with coordinates (0, 0, 1).

\[ \overrightarrow{OA} = (1, 1, 1) \]
\[ \overrightarrow{OB} = (0, 0, 1) \]

\[ A_0 = \frac{1}{2} \| \overrightarrow{OA} \times \overrightarrow{OB} \| \]
\[ = \frac{1}{2} \| \begin{vmatrix} i & j & k \\ 1 & 1 & 1 \\ 0 & 0 & 1 \end{vmatrix} \| \]
\[ = \frac{1}{2} \| i \cdot 0 - j \cdot 0 + k \cdot 1 \| \]
\[ = \frac{1}{2} \| i - j + 0 \| \]
\[ = \frac{1}{2} \sqrt{1^2 + (-1)^2} \]
\[ = \frac{1}{2} \sqrt{2} \text{ sq. units} \]
2) (i) The first three terms of a geometric sequence are $-\frac{1}{3}$, 1, -3. Determine the 5th term of the sequence.

\[
\begin{align*}
\ r &= \frac{1}{-\frac{1}{3}} = -3 \\
\ a_1 &= -\frac{1}{3} \\
\ a_2 &= 1 \\
\ a_3 &= -3 \\
\ a_5 &= 9 \\
\ a_5 &= -27.
\end{align*}
\]

(3 marks)
2) (ii) Show that 7 divides \(8^n - 1\) for all \(n \geq 1\). (7 marks)

Let \(P(n)\) be the claim that 7 divides \(8^n - 1\) for all \(n \geq 1\).

\(n = 1\), \(P(1) = 8^1 - 1 = 7\)

\(P(1)\) holds as \(7 \div 7 = 1\)

Assume \(P(k)\) holds, that is,

\[8^k - 1 = 7A, \quad A \in \mathbb{Z}^+\]

We show \(P(k+1)\) also holds

\[P(k+1) = 8^{k+1} - 1\]

\[= 8 \cdot 8^k - 1\]

\[= 7 \cdot 8^k + \underbrace{8^k - 1}_{\text{by } P(k)}\]

\[= 7 \cdot 7A + 7\]

\[= 7(8A + 1)\]

\[= 7M, \quad M \in \mathbb{Z}^+\]

Question (3) (i) see next page

\(7\) divides \(8^n - 1\) for all \(n \geq 1\).
3) (i) Solve the inequality
\[ |x-3| + |x+2| < 11 \]

(5 marks)

\[ \sqrt{A} \quad 2010 \]
3) (ii) Solve the equation

\[ |x + 1| = |x| \]  

(5 marks)

\[ \sqrt{A} \quad 2010 \]
4) (i) Determine the three values of \( z \) for which \( z^3 = 1 \). 

\[
\begin{align*}
    z^3 &= 1 \\
    z^3 &= 1 \csc \theta \\
    z^3 &= 1 \csc \left( \frac{0 + 2k\pi}{3} \right) \\
    z &= \sqrt[3]{1} \csc \left( \frac{2k\pi}{3} \right)
\end{align*}
\]

\( h = 1 \), \( z = 1 \csc \left( \frac{2\pi}{3} \right) \)

\( h = 2 \), \( z = 1 \csc \left( \frac{4\pi}{3} \right) = 1 \csc \left( \frac{-2\pi}{3} \right) \)

\( h = 3 \), \( z = 1 \csc \left( \frac{6\pi}{3} \right) = 1 \csc 0 = 1 \)

\( h = 4 \), \( z = 1 \csc \left( \frac{8\pi}{3} \right) = 1 \csc \frac{2\pi}{3} \) same as \( h = 1 \).

Question (4) (ii) see next page
4) (ii) Write \( z^4 - 16 \) as the product of linear factors and hence find all solutions of \( z^4 - 16 = 0 \) (over \( \mathbb{C} \)).

\[
z^4 - 16 = 0
\]

\[
\Rightarrow (z^2 - 4)(z^2 + 4) = 0
\]

\[
z^2 - 4 = 0 \quad \text{or} \quad z^2 + 4 = 0
\]

\[
z^2 = 4 \quad \Rightarrow z^2 = -4
\]

\[
z = \pm 2i
\]

\[
z^4 - 16 = (z - 2i)(z + 2i)(z - 2i)(z + 2i)
\]

Question (5) see next page
5) Draw the graphs and state the domain and range of each of the following functions.
(a) \( f(x) = |x| + 1 \),
(b) \( f(x) = x^2 - 1 \)
(6) Find all local maxima and minima of \( f(x) = x^3 - 2x^2 + x + 1 \) and classify them using the first derivative test. Use this information to sketch the graph of \( f \). (10 marks)

\[
f'(x) = 3x^2 - 4x + 1
\]

Critical points when \( f'(x) = 0 \)

\[
0 = 3x^2 - 4x + 1
\]

\[
0 = (3x - 1)(x - 1)
\]

\[
x = \frac{1}{3} \quad \text{or} \quad 1
\]

\[
x = \frac{1}{3} \quad f''\left(\frac{1}{3}\right) < 0.
\]

\[
x = \frac{1}{2} \quad f''\left(\frac{1}{2}\right) = +ve
\]

\[
x = 1 \quad f''(1) = 0.
\]

\[
x = 0.9 \quad f''(0.9) = -ve
\]

\[
x = 1.1 \quad f''(1.1) = +ve
\]

(\( x = 1, y = 1 \))

(\( x = \frac{1}{3}, y = \frac{1}{27} - \frac{2}{9} + \frac{1}{3} + 1 \))

(\( x = \frac{31}{27} \))
Compute the following indefinite integrals

(a) \( \int \frac{x}{x^2 + 2} \, dx \),  
(b) \( \int x^2 \cos(x^3 + 1) \, dx \)

(5 marks)

(a) Let \( u = x^2 + 2 \)

\[ \frac{du}{dx} = 2x \quad \Rightarrow \quad du = 2x \, dx \]

\[ \int \frac{x}{x^2 + 2} \, dx = \int \frac{1}{2u} \, du \]

\[ = \frac{1}{2} \ln |u| + C \]

\[ = \frac{1}{2} \ln |x^2 + 2| + C \]

(b) Let \( u = x^3 + 1 \)

\[ \frac{du}{dx} = 3x^2 \quad \Rightarrow \quad du = 3x^2 \, dx \]

\[ \int x^2 \cos(x^3 + 1) \, dx \]

\[ = \frac{1}{3} \sin(x^3 + 1) + C \]