There are two parts to this paper. Part A is worth 18 marks, and Part B is worth 42 marks. Each question in Part A is worth 1 mark for a correct answer, and 0 marks for an incorrect answer. There are 7 questions in Part B; each question carries the stated number of marks, and part marks will be awarded for correct working.

There are some formulae on the last page of this paper. You may detach this page.

Write your answers (including rough working in Part B), in the space provided. If you need extra room, use the back of other pages. You will be issued with a single sheet of blank paper. During perusal, you may write on the blank sheet, but not on this exam booklet. Do not write on the single sheet once perusal is over. Your single sheet will be collected with your exam script and will be destroyed (so material written on that sheet will not be assessed).
Part A

For each of the following 18 multiple choice questions, enter the letter corresponding to the correct answer in the corresponding box. There is no need to show any working. Each correct answer is worth 1 mark; each incorrect answer is worth 0 marks.

1. Which of the following statements is false?

(A) $-3 \in \mathbb{Z}$
(B) $\mathbb{Q} \subseteq \mathbb{R}$
(C) $\pi \in \mathbb{R}$
(D) $\{-1, -2\} \cap \{1, 2\} = \phi$
(E) $\{-1, -2\} \cup \{1, 2\} = \phi$
(F) $\{4\} \subseteq \{4, 5, 3, 1, 2\}$

Answer to Question 1: 

2. Simplify $\left( \frac{a}{b} + \frac{c}{d} \right) \div \left( \frac{a}{b} - \frac{c}{d} \right)$.

(A) $\frac{2(ad + bc)}{bd}$
(B) $\frac{ad + bc}{ad - bc}$
(C) $\frac{(ad)^2 - (bc)^2}{(bd)^2}$
(D) $-1$
(E) $0$
(F) $1$

Answer to Question 2: 

3. Let $f(x) = x^2 + 1$. Which of the following is the domain of $f(x)$?

(A) $(-\infty, \infty)$
(B) $(-\infty, 0]$  
(C) $[0, \infty)$
(D) $(0, \infty)$
(E) $(1, \infty)$
(F) $[1, \infty)$

Answer to Question 3: 

continued...
4. Let \( f(x) = x^2 + 1 \). Which of the following is the range of \( f(x) \)?

(A) \((-\infty, \infty)\)
(B) \((-\infty, 0]\)
(C) \([0, \infty)\)
(D) \((0, \infty)\)
(E) \((1, \infty)\)
(F) \([1, \infty)\)

Answer to Question 4: 

5. Which of the following is the range of the function \( f : [-2, 7) \rightarrow \mathbb{R}, f(x) = 5 - x \)?

(A) \([-2, 7)\)
(B) \((-2, 7]\)
(C) \((-2, \infty)\)
(D) \((-2, 7)\)
(E) \((-\infty, \infty)\)
(F) \([7, -2)\)

Answer to Question 5: 

6. If \( \sin \theta = \frac{\sqrt{3}}{2} \) and \( 0 \leq \theta < 2\pi \), then \( \theta \) equals which one of the following?

(A) \( \frac{\pi}{3} \)
(B) \( -\frac{\pi}{3} \)
(C) \( \frac{\pi}{6} \)
(D) \( -\frac{\pi}{6} \)
(E) \( \frac{\pi}{3} \) or \( \frac{5\pi}{6} \)
(F) \( \frac{\pi}{3} \) or \( \frac{2\pi}{3} \)

Answer to Question 6: 

continued...
7. If \( y = \log_x 1 \) where \( x \in \mathbb{R} \setminus \{0\} \), determine \( y \).

(A) 0  
(B) 1  
(C) 10  
(D) \( e \)  
(E) \( x \)  
(F) None of the above

Answer to Question 7: __________

8. If \( f(x) = 5x + 2 \), which of the following is the inverse function, \( f^{-1}(x) \)?

(A) \( f^{-1}(x) = 5x - 2 \)  
(B) \( f^{-1}(x) = -5x - 2 \)  
(C) \( f^{-1}(x) = \frac{1}{5}x + \frac{2}{5} \)  
(D) \( f^{-1}(x) = -\frac{1}{5}x - \frac{1}{2} \)  
(E) \( f^{-1}(x) = \frac{1}{5}x - \frac{2}{5} \)  
(F) \( f^{-1}(x) = -\frac{1}{5}x - \frac{2}{5} \)

Answer to Question 8: __________

9. Determine \( \lim_{x \to 1} \frac{x^2 - 1}{x - 1} \).

(A) 0  
(B) 1  
(C) 2  
(D) 3  
(E) 4  
(F) Does not exist

Answer to Question 9: __________

continued...
10. Determine \( \lim_{x \to -3} (9 - x^2) \).

(A) \(-3\)
(B) 0
(C) 3
(D) 18
(E) 4
(F) Does not exist

Answer to Question 10: \[ \square \]

11. For the graph of \( y = f(x) \) shown below, \( f'(x) \) is negative over which interval?

(A) \(-3 < x < 3\)
(B) \(-3 \leq x \leq 3\)
(C) \(x < -3 \text{ or } x > 3\)
(D) \(x \leq -3 \text{ or } x \geq 3\)
(E) \(-5 < x < 1 \text{ or } x > 4\)
(F) \(-3 < x < \infty\)

Answer to Question 11: \[ \square \]

\( \text{continued...} \)
12. For the function \( f(x) \) shown in the box, which of the following would be the best representation of \( \frac{1}{f(x)} \)?

![Graph of f(x)](image)

\[
\begin{align*}
a) \quad & \frac{1}{f(x)} \\
b) \quad & \frac{1}{f(x)} \\
c) \quad & \frac{1}{f(x)} \\
d) \quad & \frac{1}{f(x)} \\
e) \quad & f(x) = -2x^3
\end{align*}
\]

Answer to Question 12:  

[Blank]

\textit{continued...}
13. Which one of the following statements is true?

(A) \(\sin(\pi + \theta) = \cos \theta\)

(B) \(\sin^2 \theta - 1 = \cos^2 \theta\)

(C) \(\tan(-\theta) = \frac{\sin(\theta)}{\cos(-\theta)}\)

(D) \(\tan \theta \cos \theta = \sin \theta\)

(E) \(\sin\left(\frac{3\pi}{2} - \theta\right) = \sin \theta\)

(F) \(-\cos\left(\frac{\pi}{2} + \theta\right) = -\sin \theta\)

Answer to Question 13:  

14. What are the equations of the asymptotes of the graph of \(f(x) = \frac{x-2}{x+3}\)?

(A) \(x = 3, y = 1\)

(B) \(x = 3, y = -2\)

(C) \(x = 3, y = 0\)

(D) \(x = -3, y = 1\)

(E) \(x = -3, y = \frac{2}{3}\)

(F) \(x = -3, y = 2\)

Answer to Question 14:  

15. If \(y = e^{\sin x}\), which one of the following statements is true?

(A) \(\frac{dy}{dx} = e^{\sin x}\)

(B) \(\frac{dy}{dx} = e^{\sin x-1}\)

(C) \(\frac{dy}{dx} = \cos x \cdot e^{\sin x}\)

(D) \(\frac{dy}{dx} = \sin x \cdot e^{\sin x}\)

(E) \(\frac{dy}{dx} = -\sin x \cdot e^{\sin x}\)

(F) \(\frac{dy}{dx} = \cos x \cdot e^{\cos x}\)

Answer to Question 15:  

continued...
16. Which one of the following is **NOT** true about the graph of $y = \log_e x$?

(A) It has a vertical asymptote with equation $x = 0$.
(B) It passes through the point $(e, 0)$.
(C) The slope of the tangent at any point on the graph is positive.
(D) It has domain $(0, \infty)$.
(E) It has range $\mathbb{R}$.
(F) It is the reflection in $y = x$ of $f(x) = e^x$.

Answer to Question 16: __________

17. Let $p(x) = (x^2 + a)(x + b)(x - c)$, where $a$, $b$ and $c$ are three distinct positive real numbers. How many real solutions to the equation $p(x) = 0$ are there?

(A) 0
(B) 1
(C) 2
(D) 3
(E) 4
(F) 5

Answer to Question 17: __________

18. Which of the following intervals gives the solution to the inequality $|−x| < 3$?

(A) $[-2, 2]$
(B) $(-3, 3)$
(C) $(-\infty, -3) \cup (3, \infty)$
(D) $(-\infty, -3] \cup [3, \infty)$
(E) $(-3, 3) \setminus \{0\}$
(F) $[-3, 3]$

Answer to Question 18: __________

continued...
19. Each graph in the second column is the graph of the derivative of one of the functions whose graphs are in the first column. Match each graph of the function with the graph of its derivative. (4 marks)

![Graphs](image-url)

Answers to Question 19

Graph A: [ ]  Graph B: [ ]  Graph C: [ ]  Graph D: [ ]

End of Part A

Continued...
Part B

Each of the following questions carries the stated number of marks. Write your answers in the space provided. You must show full working. Part marks will be awarded for correct working.

1. Consider the function \( f(x) = (x - 1)^2(x - 2) + 1 \). If \( f'(x) = (x - 1)(ax + b) \), where \( a \) and \( b \) are constants, use differentiation to find the values of \( a \) and \( b \).

(3 marks)

continued...
2. Given that $f(-2) = -4, f(0) = 1, f'(-2) = 0, f'(x) < 0$ when $x < -2$ and $f'(x) > 0$ when $x > -2$, sketch a possible graph of the function $f(x)$. (4 marks)
3. Determine from first principles the derivative of \( f(x) = e^x \).

Note: \( f'(x) = \lim_{h \to 0} \frac{f(x + h) - f(x)}{h} \) and \( \lim_{h \to 0} \frac{e^h - 1}{h} = 1 \)
4. Let \( f(x) = \sqrt{-x + 1} \) and \( g(x) = x^2 + 2 \).

\[ \text{a. Determine } f(g(x)) \text{.} \] (1 mark)

\[ \text{b. Determine } g(f(x)) \text{.} \] (1 mark)

\[ \text{c. State the domain and range of } g(f(x)). \text{ Explain your reasoning.} \] (4 marks)
5. A model used for the number of tonnes of potatoes produced as a function of the level of nitrogen in the soil (in appropriate units) is \( P(N) = \frac{kN}{1 + N^2} \), where \( k \) is a positive constant. Find the level of nitrogen that will produce the most potatoes.

(6 marks)
6. **a.** Let \( f(x) = (\sin 2x - 3x^2)^3 \). Determine \( f'(x) \).

(3 marks)

**b.** Determine the derivative of \( y \) with respect to \( x \) if

\[
\frac{2}{x} + \ln y = 2x^4 - 5y + 3.
\]

(5 marks)

continued...
7. For the graph of \( y = 4x^3 + 27x^2 - 30x + 10 \), determine the interval for which the gradient of the graph is negative. (6 marks)
8. Find the maximum slope of the function $y = -x^3 + 3x^2 + 9x - 27$. (6 marks)

End of Exam

formulae sheet...
Some formulae:

\[
\cos(\theta + \phi) = \cos \theta \cos \phi - \sin \theta \sin \phi
\]

\[
\sin(\theta + \phi) = \sin \theta \cos \phi + \cos \theta \sin \phi
\]

\[
\cos(\theta - \phi) = \cos \theta \cos \phi + \sin \theta \sin \phi
\]

\[
\sin(\theta - \phi) = \sin \theta \cos \phi - \cos \theta \sin \phi
\]

\[
\sin(2\theta) = 2 \sin \theta \cos \theta
\]

\[
\cos(2\theta) = \cos^2 \theta - \sin^2 \theta
\]

\[
\cos(2\theta) = 2 \cos^2 \theta - 1
\]

\[
\cos(2\theta) = 1 - 2 \sin^2 \theta
\]

\[
\sin^2 \theta = \frac{1 - \cos(2\theta)}{2}
\]

\[
\cos^2 \theta = \frac{1 + \cos(2\theta)}{2}
\]