The final exam will be held on Thursday 23 June, 2005 at 2:30pm in the Exhibition Hall of the UQ Centre. You should bring:

- your student identification card;
- pencils and/or pens;
- a calculator.

The exam will have 2 hours of writing time preceded by 10 minutes of perusal. You will be provided with a blank sheet of paper in addition to the exam booklet. During perusal you may write on blank sheet of paper but not on the exam booklet. The exam paper has 17 pages and covers the entire course. The following instructions appear on the front page of the exam:

There are 100 marks on the paper. All questions carry the indicated number of marks, and students should aim to complete all questions. Page 17 is a reference page; students may detach that page. Students have been given a single sheet of blank paper and may write on that sheet during perusal (but not on the exam paper). Material written on page 17 or on the sheet of blank paper will not be assessed. Calculators are allowed, however the memory of any programmable calculator must be reset prior to the examination.

Exam preparation

Revision tutorials will be held on:

- Thursday 9 June, 10am - 1pm in building 78 room 343;
- Monday 20 June, 10am - 1pm in building 78 room 343;
- Wednesday 22 June, 12pm - 4pm in building 78 room 343.

In addition, there will be tutors available in the first year learning centre (building 67, room 443) every afternoon from 2pm - 4pm from Monday 6 June to Friday 10 June and from Tuesday 14 June to Friday 17 June.
Reference Page (this is page 17 of the 2005 final exam)

If \( A = \begin{pmatrix} a & b \\ c & d \end{pmatrix} \), then \( \det(A) = ad - bc \). If \( ad - bc \neq 0 \) then \( A^{-1} = \frac{1}{ad - bc} \begin{pmatrix} d & -b \\ -c & a \end{pmatrix} \).

If \( \mathbf{v} = \begin{pmatrix} v_1 \\ v_2 \end{pmatrix} \), then \( ||\mathbf{v}|| = \sqrt{v_1^2 + v_2^2} \) and \( \hat{\mathbf{v}} = \frac{1}{||\mathbf{v}||} \mathbf{v} \).

Cosine Rule: \( a^2 = b^2 + c^2 - 2bc \cos A \)

Sine Rule: \( \frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c} \).

Scalar product: \( \mathbf{v} \cdot \mathbf{w} = \begin{cases} 0 & \text{if } \mathbf{v} = \mathbf{0} \text{ or } \mathbf{w} = \mathbf{0}, \\ ||\mathbf{v}|| ||\mathbf{w}|| \cos \theta & \text{otherwise} \end{cases} \)

Vector product: \( ||\mathbf{v} \times \mathbf{w}|| = ||\mathbf{v}|| ||\mathbf{w}|| \sin \theta \)

Let \( \{a_n\}_{n=1}^{\infty} \) be an arithmetic sequence with first term \( a \) and common difference \( d \). Then \( a_n = a + (n - 1)d, \ (n = 1, 2, \ldots) \) and \( S_n = \frac{n}{2}(2a + (n - 1)d) \).

Let \( \{a_n\}_{n=1}^{\infty} \) be a geometric sequence with first term \( a \) and common ratio \( r \). Then \( a_n = ar^{n-1}, \ (n = 1, 2, \ldots) \), \( S_n = \frac{a(r^n - 1)}{r - 1} \) and \( \sum_{n=1}^{\infty} a_n = \frac{a}{1 - r} \) (if \( |r| < 1 \)).

If \( ax^2 + bx + c = 0 \) then \( x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \).

For \( z \in \mathbb{C} \) and \( n \in \mathbb{N} \), if \( z^n = r \cos \theta \) then \( z = r^{\frac{1}{n}} \cos \left( \frac{\theta + 2k\pi}{n} \right) \) for \( k \in \mathbb{Z} \).

The derivative of a function \( f \) at the number \( a \) is \( f'(a) = \lim_{h \to 0} \frac{f(a + h) - f(a)}{h} \).

Differentiation rules:

\[
(fg)' = fg' + gf' \quad \left( \frac{f}{g} \right)' = \frac{gf' - fg'}{g^2} \quad (f \circ g)'(x) = f'(g(x))g'(x)
\]

\[
\frac{d}{dx}(\sin x) = \cos x \quad \frac{d}{dx}(\cos x) = -\sin x \quad \frac{d}{dx}(\tan x) = \sec^2 x
\]

If \( g' \) is continuous on \([a, b]\) and \( f \) is continuous on the range of \( u = g(x) \), then

\[
\int_{a}^{b} f(g(x))g'(x) \, dx = \int_{g(a)}^{g(b)} f(u) \, du.
\]

Useful series: \( \sum_{i=1}^{n} i = \frac{n(n + 1)}{2} \) and \( \sum_{i=1}^{n} i^2 = \frac{n(n + 1)(2n + 1)}{6} \).
1. Ticket prices for a concert were $15 for adults and $12 for concession card holders. One hundred people attended the concert and ticket sales totalled $1416. Set up a pair of simultaneous equations and solve them using matrices in order to determine how many of each type of ticket were sold. (6 marks)

2. A river flows due North with a current speed of 0.2 metres per second. Barbara paddles a canoe at a speed of 0.4 metres per second in still water. She sets off from the West bank of the river and steers due East. Her canoe is blown by a wind with a speed of 0.3 metres per second from a direction of N 30° E.

Determine the resultant speed and direction of Barbara’s canoe. Round the speed to two decimal place accuracy and give the direction as a bearing to the nearest degree. (8 marks)

3. Use mathematical induction to prove that for all integers \( n \geq 1 \),

\[
\sum_{i=1}^{n} (2 + 5i) = \frac{n(5n + 9)}{2}.
\]

(8 marks)

4. Solve the following inequality. Write your solution in interval notation and illustrate it on a number line.

\[
\frac{|x - 2|}{3 + x} \leq 4
\]

(Hint: You may want to consider three cases, based on where the numerator and denominator of the fraction change sign.) (7 marks)

5. Let \( \mathbf{u} \) be the vector with magnitude \( 2\sqrt{2} \) and direction \( \frac{7\pi}{4} \). Let \( \mathbf{v} = -\sqrt{3}\mathbf{i} - \mathbf{j} \). Determine \( \mathbf{u} \cdot \mathbf{v} \). (5 marks)

6. Determine all values of \( z \in \mathbb{C} \) such that \( z^3 = i \). Illustrate your values of \( z \) on an Argand diagram. (5 marks)

7. Let \( f(x) = \sqrt{x^2 - 4} \) and let \( g(x) = \frac{1}{x} \).

(a) State the domain and range of \( f \). (2 marks)

(b) State the domain and range of \( g \). (2 marks)

(c) Determine \( (g \circ f)(x) \) and state its domain and range. (3 marks)

(d) Is \( f \) a one-to-one function? Explain your answer. (2 marks)

8. Let \( f(x) = \sqrt{x - 1} \).

(a) Use the definition of the derivative to calculate \( f'(2) \). (5 marks)

(b) Determine the equation of the tangent line to \( f \) at \( x = 2 \). (2 marks)
9. Determine the derivative of each of the following functions.
   (a) \( f(x) = 3x^3 - 2x + 3 + \frac{2}{x} \)  \hspace{1cm} (b) \( g(u) = (4u^3) \tan u \)  \hspace{1cm} (2 marks)  \hspace{1cm} (2 marks)
   (c) \( f(\theta) = \cos(2\theta^2 - \theta) \)  \hspace{1cm} (d) \( g(x) = \frac{e^{2x}}{x} \)  \hspace{1cm} (2 marks)  \hspace{1cm} (2 marks)

10. Let \( f(x) = -x^3 + 3x^2 + 16 \).
   (a) Determine \( f'(x) \) and \( f''(x) \).  \hspace{1cm} (2 marks)
   (b) Identify all intervals on which \( f \) is increasing and all intervals on which \( f \) is decreasing. \hspace{1cm} (3 marks)
   (c) Determine and classify all local maxima and minima of \( f \). \hspace{1cm} (2 marks)
   (d) Identify all intervals on which \( f \) is concave up and all intervals on which \( f \) is concave down. \hspace{1cm} (3 marks)
   (e) One solution of \( f(x) = 0 \) is \( x = 4 \). Show that there are no other real solutions to \( f(x) = 0 \). \hspace{1cm} (5 marks)
   (f) Use the information in parts (a) to (e) to sketch a graph of \( f \). \hspace{1cm} (You may use unequal scales on the axes.) \hspace{1cm} (1 mark)

11. Determine the following indefinite integrals.
   (a) \( \int (3x^2 - 5x) \, dx \)  \hspace{1cm} (2 marks)
   (b) \( \int \frac{2x - 1}{x^2 - x} \, dx \)  \hspace{1cm} (2 marks)
   (c) \( \int 6\theta^2 \sin(\theta^3) \, d\theta \)  \hspace{1cm} (3 marks)

12. Let \( f(x) = -x^2 + 4x \).
   (a) Evaluate the Riemann sum for \( f(x) \), with \( 0 \leq x \leq 4 \), having 40 subintervals and taking the sample points to be the right endpoints. (Hint: You might like to use the useful series from the reference page.) \hspace{1cm} (6 marks)
   (b) Evaluate the definite integral \( \int_0^4 f(x) \, dx \).  \hspace{1cm} (2 marks)

13. A car is travelling at 72 km/h (that is 20 m/s) when the brakes are applied, producing a constant deceleration of 7 m/s\(^2\) until the car stops. Determine the distance travelled from the moment the brakes are applied to when the car stops. Round your answer to the nearest metre. \hspace{1cm} (6 marks)