Work through the following problems, show your tutor then record your name before the end of your Week 4 tutorial. You are encouraged to discuss these questions and your solutions with your peers and to ask your tutor for assistance. Working through ten sets of tutorial problems is compulsory and each of the ten problem sets will contribute 1% towards your final grade. Note that you earn the 1% for your effort in solving these problems during the tutorial rather than for answering all the problems correctly.

Once you have finished these problems, you can use the remainder of your tutorial time to work on other aspects of the course. Solutions to the tutorial problems will be distributed next week.

1. Let $C = \begin{pmatrix} 1 & 2 & 1 \\ 1 & 1 & 2 \\ 1 & 3 & -1 \end{pmatrix}$.

   a) Show by direct multiplication that $C^{-1} = \begin{pmatrix} -7 & 5 & 3 \\ 3 & -2 & -1 \\ 2 & -1 & -1 \end{pmatrix}$.

   b) Use the result from part (a) to solve the following system of equations.

      $x + 2y + z = -1$
      $x + y + 2z = -1$
      $x + 3y - z = 2$

2. Determine for which values of $x$ the following determinants are equal to zero.

   $\begin{vmatrix} 1-x & 2 \\ 3 & 2-x \end{vmatrix} \begin{vmatrix} 1 & 1 & 4 \\ 2 & x & 1 \\ 2x & x^2 & x \end{vmatrix} \begin{vmatrix} 1 & 2 \\ x^2 & x \end{vmatrix}$

3. Entry charges for The Magic of Maths is $25 for adults and $15 for children. A party of 21 people pays $375 in total. Use matrices to determine how many children there were in the party.

4. a) Write down the matrix form of the vectors $a$ and $b$ illustrated below.
b) Draw the vectors \( \mathbf{u} = \begin{pmatrix} -1 \\ 2 \end{pmatrix} \) and \( \mathbf{v} = \begin{pmatrix} 3 \\ -2 \end{pmatrix} \) on the axes below.

5.a) Let \( \mathbf{a} = \begin{pmatrix} 3 \\ -2 \\ 5 \end{pmatrix} \) and \( \mathbf{b} = \begin{pmatrix} -2 \\ 4 \\ 8 \end{pmatrix} \).

Determine the vectors \( \mathbf{c} \) and \( \mathbf{d} \) in matrix form, where

\[ \mathbf{c} = 3\mathbf{a} + \frac{1}{2}\mathbf{b} \quad \text{and} \quad \mathbf{d} = -2\mathbf{b} - \mathbf{a}. \]

b) Let \( \mathbf{u} \) and \( \mathbf{v} \) be the vectors illustrated below.

Use geometric vector addition and scalar multiplication to illustrate the vectors

\[ \mathbf{c} = 2\mathbf{u} + \mathbf{v} \quad \text{and} \quad \mathbf{d} = \mathbf{v} - \mathbf{u}. \]