1. Since it is a G.S.

\[
\frac{x}{(3-x)} = \frac{2-x}{x}
\]

\[
x^2 = (2-x)(3-x)
\]

\[
x^2 = 6 - 5x + x^2
\]

\[
5x = 6
\]

\[
x = \frac{6}{5}
\]

The first three terms of the G.S are \(\frac{9}{5}, \frac{6}{5}, \frac{4}{5}\) and so the common ratio is \(\frac{2}{3}\).

2. We obtain the equations

\[
\frac{y}{x} = \frac{z}{y} \quad (1)
\]

\[
x + y + z = 9 \quad (2)
\]

\[
x - y = z - x \quad (3)
\]

From Equation 3 we obtain \(2x - z = y\) and when we substitute this into Equation 2 we obtain

\[
x + (2x - z) + z = 9
\]

\[
3x = 9
\]

\[
x = 3. \quad (4)
\]

Substituting into Equation 1 we obtain

\[
\frac{6-z}{3} = \frac{z}{6-z}
\]

\[
(6-z)^2 = 3z
\]

\[
36 - 12x + z^2 = 3z
\]

\[
z^2 - 15z + 36 = 0
\]

\[
(z - 12)(z - 3) = 0
\]

So \(z = 12\) or \(3\) and \(y = -6\) or \(3\).

Thus we have that the three successive terms of the G.S. are \(3, -6, 12\) and of the A.S. are \(-6, 3, 12\) or for G.S. \(3, 3, 3\) and A.S. \(3, 3, 3\). Note that in the second case the common ratio is 1 and the common difference is 0. While this may be a boring sequence it is a sequence.

3. The sequence of powers of three \(1, 3, 9, \ldots, 4782969\) is a geometric sequence having 15 terms, with first term \(a = 1\) and common ratio \(r = 3\). Thus, the corresponding series has sum

\[
1 + 3 + 9 + \cdots + 4782969 = \frac{1(3^{15}-1)}{3-1} = 7174453.
\]

4. Let \(P(n)\) be the statement \(\sum_{j=1}^{n} 2^{j-1} = 2^n - 1\).

**Step 1:** We need to show this statement is true for \(n = 1\). L.H.S of \(P(1)\) is \(\sum_{j=1}^{1} 2^{j-1} = 2^1 = 2^0 = 1\). The R.H.S of \(P(1)\) is \(2^1 - 1 = 2 - 1 = 1\). The L.H.S equals the R.H.S. so the statement is true for \(n = 1\).
Step 2: Assume the statement is true for \( n = k \), so assume \( \sum_{j=1}^{k} 2^{j-1} = 2^k - 1 \).

Step 3: Prove the statement is true for \( n = k + 1 \), that is, prove \( \sum_{j=1}^{k+1} 2^{j-1} = 2^{k+1} - 1 \).

L.H.S. of \( P(k+1) = \sum_{j=1}^{k+1} 2^{j-1} = \sum_{j=1}^{k} 2^{j-1} + 2^{k+1-1} \)

\[ = 2^k - 1 + 2^k \quad \text{since it is true for } n = k \]

\[ = 2(2^k) - 1 \]

\[ = 2^{k+1} - 1 \]

Therefore, if the statement is true for \( n = k \), then it is true for \( n = k + 1 \). Hence, by mathematical induction \( \sum_{j=1}^{n} 2^{j-1} = 2^n - 1 \) for all integers \( n \geq 1 \).

5.

(a) \( c(x) = \sqrt{3 - x} \)

Since we cannot have the square root of a negative number, we need \( 3 - x \geq 0 \). The domain of \( c \) is \( (-\infty, 3] \) and the range of \( c \) is \([0, \infty)\).

(b) \( d(x) = \frac{2}{x + 1} \)

Since we cannot divide by zero, we need \( x + 1 \neq 0 \). The domain of \( d \) is \( \mathbb{R} \setminus \{-1\} \) and the range of \( d \) is \( \mathbb{R} \setminus \{0\} \).

(c) \( f(x) = \ln(x + 1) \)

Since we cannot have the logarithm of a negative number or zero, we need \( x + 1 > 0 \). The domain of \( f \) is \( (-1, \infty) \) and the range of \( f \) is \( \mathbb{R} \).

(d) \( h(x) = \begin{cases} 
2 - x & \text{if } x \geq 1 \\
3 & \text{if } x < 1
\end{cases} \)

The domain of \( h \) is \( \mathbb{R} \) and the range of \( h \) is \( (-\infty, 1] \cup \{3\} \).