Complete the following problems and submit your solutions into the assignment box for your tutorial group. The assignment boxes are located on level three of the Priestley (Mathematics) Building (67), next to the lifts. The boxes are clearly labelled as MATH1050, and there is one for each of the tutorial groups T1-T19. Please take care to ensure your assignment is placed in the correct box.

Late assignments will be accepted only with a medical certificate or evidence of other exceptional circumstances.

Ensure that your assignment is stapled, and that your name, student number, tutor’s name and tutorial group are clearly marked on the front page of your assignment before you hand it in. You do NOT need a cover sheet.

Each of the five assignments will contribute 3% towards your final grade. Although you are encouraged to discuss the assignment questions with your peers, your written assignment must be your own work. Solutions to the assignment will be distributed approximately one week after the due date.

1. Let $A = \begin{pmatrix} x & 3 \\ 4 & 4 + x \end{pmatrix}$. For what values of $x$ is the matrix singular?

2. Let $B = \begin{pmatrix} \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \end{pmatrix}$. Calculate $B^{100}$.

3. Let $A = \begin{pmatrix} 2 & 4 & 0 \\ 0 & 2 & 2 \\ 2 & 0 & -2 \end{pmatrix}$ and $B = \begin{pmatrix} -1 & 2 & 2 \\ 1 & -1 & -1 \\ -1 & 2 & 1 \end{pmatrix}$.
   a) Calculate det($A$) and det($B$)
   b) Calculate $AB$
   c) Use the result from part (b) to solve the following system of equations.

$$
-x + 2y + 2z = 1 \\
x - y - z = 0 \\
x + 2y + z = -1
$$
4. Three forces are acting on an object in 2-space. The first force is 25N and is at an angle of 0° to the horizontal. The second force is 30N and the third force is 40N. The three forces are at angles of 120° to each other. What is the resultant force acting on the object and in which direction is it? Give all possible solutions.

5. Let \( v = (-\cos \theta, \sin \theta) \) and \( w = (\cos \theta, -\sin \theta) \). Find \( v \cdot w \).

6. A(-1, 2, -2), B(2, 14, 2), C(2, -3, 4) and D(6, -3, 7) are four points in space. Find the angle (in both degrees and radians) between AB and CD.

7. Calculate the area of the triangle ACB if the coordinates are (2, 1, 0), (3, 2, 1) and (4, -2, 2) respectively.