1. (a) First we number the equations for convenience.

\[-x - 5y = -29 \quad (1)\]
\[5x - y = -15 \quad (2)\]

We solve these using substitution. Rearranging equation (1) gives

\[x = -5y - 29 \quad (3)\]

Substituting equation (3) into equation (2) gives

\[5(-5y - 29) - y = -15. \quad (4)\]

Then expanding the brackets in equation (4) gives

\[-25y - 145 - y = -15, \quad (5)\]

and simplifying equation (5) gives

\[-26y - 145 = -15. \quad (6)\]

Then solving equation (6) gives \(-26y = 130\) so \(y = \frac{130}{-26} = -5\).

Next we substitute the value for \(y\) into equation (3) to obtain the value for \(x\), giving

\[x = -5 \times -5 - 29 \quad \text{so} \quad x = -4.\]

Hence the simultaneous solution to equations (1) and (2) is \(x = -4\) and \(y = -5\).

(As always, check your answers by substituting into equations (1) and (2).
(1) \(-x - 5y = -1 \times -4 - 5 \times -5 = 4 + 25 = 29\), as required.
(2) \(5x - y = 5 \times -4 - 1 \times -5 = -20 + 5 = -15\), as required.

Hence the answer is correct.)

(b) First we number the equations for convenience.

\[3x - y = -5 \quad (1)\]
\[-3x + y = -3 \quad (2)\]

Now we add both sides of equations (1) and (2), giving

\[3x - 3x - y + y = -5 - 3. \quad (3)\]

Simplifying equation (3) gives

\[0 + 0 = -8. \quad (4)\]

Then solving equation (4) gives \(0 = -8\).

This statement is never true, so there is no solution to the given equations.

(c) First we number the equations for convenience.

\[-5x - y = 7 \quad (1)\]
\[-5x - y = -7 \quad (2)\]

We solve these using elimination. Multiply equation (1) by \(-1\), giving

\[5x + y = -7 \quad (3)\]
\[-5x - y = -7 \quad (4)\]

Now we add both sides of equations (3) and (4), giving

\[5x - 5x + y - y = -7 - 7. \quad (5)\]

Simplifying equation (5) gives

\[0 + 0 = -14. \quad (6)\]

Then solving equation (6) gives \(0 = -14\).

This statement is never true, so there is no solution to the given equations.
(d) First we number the equations for convenience.

\[-7x + 6y = -7 \quad (1)\]
\[7x - 6y = 7 \quad (2)\]

Now we add both sides of equations (1) and (2), giving

\[-7x + 7x + 6y - 6y = -7 + 7. \quad (3)\]

Simplifying equation (3) gives

\[0 + 0 = 0. \quad (4)\]

Then solving equation (4) gives \[0 = 0.\]

This statement is always true, so there is an infinite number of solutions to the given equations.

\[\text{\underline{2. (a)}}\]

(a) First we number the equations for convenience.

\[-2x + 2y = 4 \quad (1)\]
\[-5x - 5y = 30 \quad (2)\]

Next we take out any common factors.

\[-x + y = 2 \quad (3)\]
\[-x - y = 6 \quad (4)\]

Now we add both sides of equations (3) and (4), giving

\[-x - x + y - y = 2 + 6. \quad (5)\]

Simplifying equation (5) gives

\[-2x + 0 = 8. \quad (6)\]

Then solving equation (6) gives \[-2x = 8\] so \[x = \frac{8}{-2} = -4.\]

Next we substitute the value for \(x\) into equation (3) to obtain the value for \(y\), giving

\[-1 \times -4 + y = 2 \quad \text{so} \quad 1y = -2.\]

Hence the simultaneous solution to equations (1) and (2) is \(x = -4\) and \(y = -2.\)

(As always, check your answers by substituting into equations (1) and (2).

(1) \(\rightarrow -2x + 2y = -2 \times -4 + 2 \times -2 = 8 - 4 = 4\), as required.

(2) \(\rightarrow -5x - 5y = -5 \times -4 - 5 \times -2 = 20 + 10 = 30\), as required.

Hence the answer is correct.)

(b) First we number the equations for convenience.

\[-2x - y = 5 \quad (1)\]
\[x + 5y = -5 \quad (2)\]

We solve these using substitution. Rearranging equation (1) gives

\[y = -2x - 5 \quad (3)\]

Substituting equation (3) into equation (2) gives

\[5(-2x - 5) + x = -5. \quad (4)\]

Then expanding the brackets in equation (4) gives

\[-10x - 25 + x = -5, \quad (5)\]

and simplifying equation (5) gives

\[-9x - 25 = -5. \quad (6)\]

Then solving equation (6) gives \(-9x = 20\) so \[x = \frac{20}{-9} = -\frac{20}{9}.\]
Next we substitute the value for $x$ into equation (3) to obtain the value for $y$, giving

$$y = -2 \times \frac{-20}{9} - 5 \quad \text{so} \quad y = \frac{-5}{9} = \frac{-5}{9}.$$ 

Hence the simultaneous solution to equations (1) and (2) is $x = \frac{20}{9}$ and $y = \frac{-5}{9}$.

(As always, check your answers by substituting into equations (1) and (2).)

(1) $\rightarrow -2x - y = -2 \times \frac{20}{9} - 1 \times \frac{-5}{9} = \frac{40}{9} + \frac{5}{9} = 5$, as required.

(2) $\rightarrow x + 5y = 1 \times \frac{20}{9} + 5 \times \frac{-5}{9} = \frac{-20}{9} - \frac{25}{9} = -5$, as required.

Hence the answer is correct.)

(c) First we number the equations for convenience.

$$4x - 7y = -3 \quad (1)$$
$$4x - 7y = -6 \quad (2)$$

We solve these using elimination. Multiply equation (1) by $-1$, giving

$$-4x + 7y = 3 \quad (3)$$
$$4x - 7y = -6 \quad (4)$$

Now we add both sides of equations (3) and (4), giving

$$-4x + 4x + 7y - 7y = 3 - 6 \quad (5)$$

Simplifying equation (5) gives

$$0 + 0 = -3. \quad (6)$$

Then solving equation (6) gives $0 = -3$.

This statement is never true, so there is no solution to the given equations.

(d) First we number the equations for convenience.

$$6x + 4y = -2 \quad (1)$$
$$3x + 2y = -1 \quad (2)$$

Next we take out any common factors.

$$3x + 2y = -1 \quad (3)$$
$$3x + 2y = -1 \quad (4)$$

We solve these using elimination. Multiply equation (3) by $-1$, giving

$$-3x - 2y = 1 \quad (5)$$
$$3x + 2y = -1 \quad (6)$$

Now we add both sides of equations (5) and (6), giving

$$-3x + 3x - 2y + 2y = 1 - 1 \quad (7)$$

Simplifying equation (7) gives

$$0 + 0 = 0. \quad (8)$$

Then solving equation (8) gives $0 = 0$.

This statement is always true, so there is an infinite number of solutions to the given equations.
3. (a) 2 points on the track are \((x_1, y_1) = (-3, 10)\) and \((x_2, y_2) = (1, 2)\). So \(m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{2 - 10}{1 + 3} = \frac{-8}{4} = -2\). Hence \(y = -2x + c\). Now \((1, 2)\) on the line, so 
\[2 = -2 \times 1 + c \implies c = 4 \implies y = -2x + 4.\]

(b) Points \((x_1, y_1) = (-1, 6)\) and \((x_2, y_2) = (-3, 2)\). So \(m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{2 - 6}{-3 + 1} = \frac{-4}{-2} = 2\). Hence \(y = 2x + c\). Now \((-3, 2)\) on the line, so \(2 = 2 \times -3 + c\) 
\[\implies 2 = -6 + c \implies c = 8 \implies y = 2x + 8.\]

(c) If Sassie’s ride is parallel to Dagwood’s, then the slope of Sassie’s ride must also have slope \(m = 2\). So \(y = 2x + c\). Now Sassie starts at \((-2, 0)\), so \(0 = 2 \times -2 + c \implies c = 4\), so \(y = 2x + 4\).
To find the crossing point, 2 equations: \(y = 2x + 4\) (Sassie) and \(y = -2x + 4\) (train).
Hence \(2x + 4 = -2x + 4 \implies 4x = 0 \implies x = 0\). Therefore \(y = 2 \times 0 + 4\), so \(y = 4\). Sassie crosses the railway line at \((0, 4)\).

4. (a) Let \(x = \#\) whiskeys and \(y = \#\) wiener.
So \(y = x + 3\) \((1)\) and \(6x + 2y = 54\) \((2)\)
Substitute (1) into (2), \(6x + 2(x + 3) = 54 \implies 6x + 2x + 6 = 54 \implies 8x = 48 \implies x = 6\) and \(y = 9\). She buys 6 whiskeys and 9 wiener. Check!

(b) i) \(x + y + z = 24\) \((1)\) \[y = \frac{1}{3}(x + z)\] \((2)\) \[z = 2x\] \((3)\)

ii) Now these are 3 equations and 3 unknowns, but we solve in the usual way. Use (3) to substitute into (1) and (2):
\( (1) \implies 2x + y + z = 24 \implies y + 3z = 24\) \((4)\)
\( (2) \implies y = \frac{1}{3}(x + 2z) = \frac{1}{3}(3x) = x \implies y = x\) \((5)\)
Now substitute (5) into (4) \(\implies x + 3x = 24 \implies 4x = 24 \implies x = 6\). Therefore from (5) \(y = 6\). Substitute \(x = 6\) and \(y = 6\) into (1) \(\implies x + 6 + 6 = 24 \implies x = 12\).

She spends 12 hours studying, 6 hours sleeping and 6 hours writing poetry.

Finally, you should check these answers by substituting into the three original equations.
When you do so, you'll see that they are all true.