MATH1040 Assignment 7

All questions should be submitted by 5pm on Thursday 25 January. You should show full working where possible. Assignments can be submitted during your tutorial or to the MATH1040 assignment box on the 3rd floor of the Priestley Building (#67). Make sure that your name and student number are on each sheet of your answers. Solutions will be distributed in class later.

1. If \( f(x) = 3^x \), \( g(x) = 2^x \) and \( h(x) = 2^x \), evaluate the following:
   a) \( f(2) \)  
   b) \( g(3) \)  
   c) \( g(-2) \)  
   d) \( f(-4) \)  
   e) \( h(3) \)  
   f) \( h(-2) \)

2. Evaluate the following:
   a) \( \log_3 9 \)  
   b) \( \log_9 3 \)  
   c) \( \log_3 1 \)  
   d) \( \ln 3 \)  
   e) \( \ln 0 \)

3. Find \( y' \) for each of:
   a) \( y = 4x \)  
   b) \( y = -3x^4 + 5x^3 - 6x + 4 \)  
   c) \( y = 3x^2 + 4x^3 \)  
   d) \( y = 2e^{3x} - e^{-2x} \)  
   f) \( y = 2 \ln x \)

4. Find \( y' \) for each of:
   a) \( y = e^{x^2} \)  
   b) \( y = x^2 \ln x \)  
   c) \( y = (x^4 + 3)^5 \)  
   d) \( y = (2e^{3x} - e^{-2x})^3 \)

5. Let \( y = -x^2 + x + 1 \).
   (a) Find the derivative of \( y \). (Previous exam question.)
   (b) Using your answer to Part (a), find the slope of \( y \) at the points \((1, 1), (1/2, \sqrt{3}/4)\) and \((0, 1)\).
   (c) Find the \( x \) coordinates of points at which the value of the function equals the slope of the function.

6. The Beijing Olympic games look like being a disaster. In an effort to improve the broad general appeal of the games, it has been decided to introduce a new sport: nude stunt pig riding. Contestants have to try to ride their pigs on a difficult course, shaped much like a roller-coaster track, without falling off. Pablo is a member of the Puerto Rican pig riding team.

   (a) The first stage of competition involves riding on a track which matches the equation \( y = x^2 - 6x + 6 \), where \( x \) represents horizontal distances in metres, and \( y \) represents the height of the track, in metres.
      (i) Find the slope of the track at any point (that is, find the derivative of \( y \)).
      (ii) Pablo will fall off his pig if the track has a slope more than 8 or less than -4. Find the \( x \) coordinates of the points at which the slope of the track equals 8 or -4.
      (iii) Write in interval form the values of \( x \) between which Pablo can ride without falling off.
      (iv) What is the largest height Pablo and his pig can attain on the track? (Hint: this will happen at one of the points found in (ii). Calculate the \( y \)-values at each point.)
      (v) Pablo remarks that nude pig riding is quite enjoyable, except for some discomfort where the pig is exactly level (because at that point the slope is changing from a negative slope to a positive slope, or positive to negative, and the changeover hurts!). At what value of \( x \) is Pablo's pig exactly level?

   (b) The second stage of competition involves a much harder course, which matches the equation \( y = \frac{1}{2}x^3 + x^2 - 24x + 4 \).
      (i) Find out where Pablo and his pig are exactly level?
      (ii) Alas! There is controversy over the uniform in this sport! Pablo cheats and attaches a small piece of velcro to himself and his pig. Now he can stay on until the track has slope 11. At what values of \( x \) does the track have this slope?
BONUS QUESTIONS (3 marks, 4 marks, 8 marks)

1. How much money would need to be invested now in order to have $2000 in two years' time if it is invested in an account paying 6.8% interest p.a. compounding:
   a) monthly?
   b) continuously?
   (Assume there are no fees.)

2. The sum of one number and three times a second is 30. Using calculus (i.e. derivatives), find which particular numbers have a product which is as large as possible.

3. A team of police investigators is at a jungle grave site 180m north of their cars, which are all parked on a dirt road running roughly east-west. When an important forensic clue is found at the grave site, the fittest member of the team, Wally, is dispatched to telephone details to headquarters from the nearest transmitting spot which is 2km west of the cars, along the dirt road. If Wally can manage an average of 3km/hr in the jungle and 6km/hr once he makes it to the road, at what point west of the cars, on the dirt road, should he aim to emerge from the jungle if he intends to reach the transmitting site as soon as possible.