MATH1040 Assignment 6

All questions should be submitted by 6pm on Wednesday 2 May. You should show full working where possible. Assignments are to be submitted during your tutorial. Make sure that your name and student number are on each sheet of your answers. Solutions will be distributed in class later. There is a bonus question at the end of the assignment.

1. Temperatures on the Fahrenheit and Celsius scales are related by the equation $5F - 9C = 160$.
   (a) Water freezes at $0^\circ C$. What is this temperature in Fahrenheit?
   (b) Some people convert from Celsius to Fahrenheit in their heads by first multiplying the Celsius figure by 1.8, then adding 32. Explain why this works. (Hint: rearrange the initial equation.)
   (c) If you place your tongue on a metal pole at a temperature of $-20^\circ C$, then your tongue will immediately freeze, and stick to the pole. What is this temperature in Fahrenheit?
   (d) At what temperature will a thermometer read the same number on both scales?
   (e) In a previous assignment we saw that $t = \frac{n}{8} + 6$ is a formula for finding the temperature (in Celsius) based on the number of chirps a cricket makes per minute. Rewrite this formula so that it finds the temperature in Fahrenheit.

2. The evil Dr Moriarty has tied Penelope Pureheart to the railway line. The 11:05 express train is racing towards Penelope along the line $y = 3x - 2$. Penelope's boyfriend Dagwood Doogood is at the point $(2, 7)$, sitting on his horse.
   (a) Dagwood must ride in a straight line toward the railway line and cross the line at the point $(-1, -5)$ in front of the train, forcing it to stop before it strikes Penelope. What is the equation of the straight line marking Dagwood's ride?
   (b) Unfortunately Dagwood is a twit (Penelope does not love him for his mind). He has no sense of direction, so needs help from Sassie the wonder dog. Sassie starts at the point $(2, 5)$ and travels toward the railway track on a line parallel to Dagwood's. At what point will Sassie cross the railway line? (Hint: you'll need the equations for each of the train, Dagwood and Sassie.)

3. Like most MATH1040 students, Sarina always falls asleep in lectures. She dreams of Norwegian men; she adores Norwegian men. This week, she dreams of Luscious Lars. Sadly, due to an unfortunate incident in his childhood, Luscious Lars is obsessed with maths and moose, and doesn't notice Sarina.
   (a) To attract his attention, Sarina invites Lars to a wild night out at Murwillumbah's massive moose-and-maths event, "Moothematica", to woo him with wiener and whisky. Each whisky costs $5, and each wiener costs $3. However, if she drinks too much she'll pass out, leaving Luscious Lars to run off with a moose handler (or worse, do some maths), so the number of wiensers must be exactly 4 more than the number of whiskies. If she spends $44, how many of each item does she buy?
   (b) Alas! The moose-and-maths event did not work. Sarina decides to write Lars some love poetry. There are 30 hours left before the MATH1040 exam; Sarina decides to spend all the remaining time studying, watching "The Biggest Loser" and writing poetry. However, because she wants to pass (and has slept through all of her lectures), she needs three times as much time studying as she spends writing poetry. She also spends one half as much time watching "Loser" as she spends on all other activities.
   (i) Write three equations outlining Sarina's timing dilemma. (Hint: Let $x$ be the amount of time she spends studying, $y$ be the amount of time she spends watching "Loser" and $z$ be the amount of time she spends writing poetry.)
   (ii) Solve these equations, thus giving how much time she should spend on each activity. (Hint: you now have three equations with three unknowns. Solve them using substitution.)

(continued over...
4. If \( f(x) = x - x^2 \),
   
   (a) Find \( f(3) \).
   (b) Find \( f(-2) \).
   (c) Let \( g(x) = x^2 \). Find \( f(g(x)) \).
   (d) Let \( g(x) = (x + h) \). Find \( f(g(x)) \).

5. Answer each of the following questions.
   
   (a) Find the domain of \( f(x) = \sqrt{2 + x} \)
   (b) Find the domain of \( f(x) = \frac{11}{x + 1} \)
   (c) Find the domain of \( f(x) = 4 + |x| \)
   (d) Find the range of \( f(x) = -x^2 + 8 \)
   (e) Find the range of \( f(x) = \sqrt{-12x} \)
   (f) Find the range of \( f(x) = 4 + |x| \)

BONUS QUESTION (5 marks)

The first two scales below are in perfect balance. How many spades (on the right-hand side) will be needed to balance the bottom scale?