MATH1040 Assignment 4

All questions should be submitted by 6pm on Wednesday 28 March. You should show full working where possible. Assignments are to be submitted during your tutorial. Make sure that your name and student number are on each sheet of your answers. Solutions will be distributed in class later.

1. For the following questions let \( r_1 \) and \( r_2 \) be random natural numbers chosen independently, where \( r_1 \) is between 1 and 10 (inclusive), and \( r_2 \) is between 1 and 8 (inclusive). In each case, find the probability \( p \) that:
   
   (a) \( r_1 \) is even?
   (b) \( r_1 \geq 5 \)?
   (c) \( r_1 \) is even and \( r_1 \geq 5 \)?
   (d) \( r_1 \) is even or \( r_1 \geq 5 \)?
   (e) \( r_1 \) is even given that \( r_1 \geq 5 \)?
   (f) Both \( r_1 \) and \( r_2 \) are even?
   (g) At least one of \( r_1 \) and \( r_2 \) is even?
   (h) \( r_1 \) is even given that \( r_2 \) is even?

2. Chaps find Rebecca irresistible, and keep on asking her out. She gets sick of it, and creates a list of excuses for not going out with them. Each time someone asks her out, she picks an excuse at random from her list, and uses that excuse to avoid them. Today, three particularly hideous mathematicians (Boris, Noris and Horace) ask her out.

   (a) If there are four excuses on her list, and Boris asks her out twice, what is the probability that Rebecca will use exactly the same excuse both times?
   
   (b) Three of her excuses are good (for example “I’ve caught a virulent strain of Venezuelan Beaver Blight, so I’m too sick” or “I’ve been grounded indefinitely for grossly mistreating a hamster”), but the other one is weak (“I want to study MATH1040”).
      
      (i) If Noris asks her out once, what is the probability that she will use a weak excuse?
      (ii) If he asks her out twice, what is the probability that she will use two good excuses?
   
   (c) Some chaps ask her out twice, so she creates a list of possible pairs of excuses. However, she also decides to modify her excuse strategy. No pair should use the same excuse twice, and she should never use a weak excuse after using a good one. Assuming she selects at random from the list of allowable pairs of excuses and that Horace asks her out twice: (Hint: draw a table of possible outcomes, showing all allowable pairs of excuses.)
      
      (i) What is the probability that exactly one of her excuses is weak?
      (ii) What is the probability that both of her excuses are weak?
      (iii) What is the probability that neither of her excuses is weak?

3. A group of witches has moved into the administration building at the University. Each witch has one or more magical powers: the powers are to turn people into toads, make people’s hair fall out, and to fly on broomsticks (we will call these powers toad, hair and fly).

(continued over...)

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There are 26 witches in total. All up, seventeen witches have the toad power, fifteen can fly and ten have the hair power. Eight witches who can fly also have the toad power. Three witches have all three powers. The number of witches with only the hair power equals the number of witches with the hair and toad powers (but who cannot fly). Six witches can both fly and have the hair power (some may also have the toad power).

(a) Represent the witches using a Venn diagram with three sets (one set $T$ being witches with toad power, one set $H$ witches with hair power and one set $F$ witches with fly power). Instead of writing the elements on each region of the sets, write the number of elements which occur in each region. To get you started, write 3 in the region corresponding to $T \cap H \cap F$, as there are three witches with all three powers.

(b) If a witch is chosen at random, what is the probability that her only power is flying?

(c) If a witch is chosen at random, what is the probability that she can fly, given that she has the toad power?

(d) In a regular competition there is a randomly chosen Women’s Weekly Witch of the Week. What is the probability that all of the first three winners each have exactly one power? (Assume each selection is independent of all other selections, so the same witch could win more than once.)

(e) The Vice-Chancellor is worried about the witches (for example, he doesn’t want his lovely new hair-do ruined, and the air-traffic controllers have complained about low-flying broomsticks), so he asks Buffy the Vampire Slayer to help solve the problem. She arrives on a Tuesday night. However, on Tuesday nights, all the witches with just the toad power attend a meeting at the frog-lovers’ society, so they are absent when she arrives. She randomly selects one of the remaining witches, and kills the witch by plunging a stake in her heart. What is the probability that the dead witch could fly but did not have the hair power?

4. Write a 150 word essay on the Monty Hall problem. (You need to write more than a summary of what we did in class. You may use the internet to locate material if you like. Remember the strict University rules against plagiarism: you must write your explanations in your own words. If you like, you can do an experiment on your friends and your essay can be a presentation of the results.)

5. Arrange each of the following events into rough decreasing order of probability. (You may like to simply write the corresponding letters (a) to (g) in decreasing order of probability as your answer.)

(a) A family of three children being all girls.
(b) Winning Gold Lotto Division 1 with one entry.
(c) An adult Australian dying in a car crash this year.
(d) Tossing a coin 20 times and getting all Heads.
(e) In a group of 20 people, at least 2 people having the same birthday.
(f) An Australian soldier fighting at Gallipoli being killed.
(g) Winning Powerball Division 1 if you play 1,000 games each week for a year.